XIO RATIO

and 26, the Transdimensional Number

Add 26% to the edge of a cube and you about double its volume

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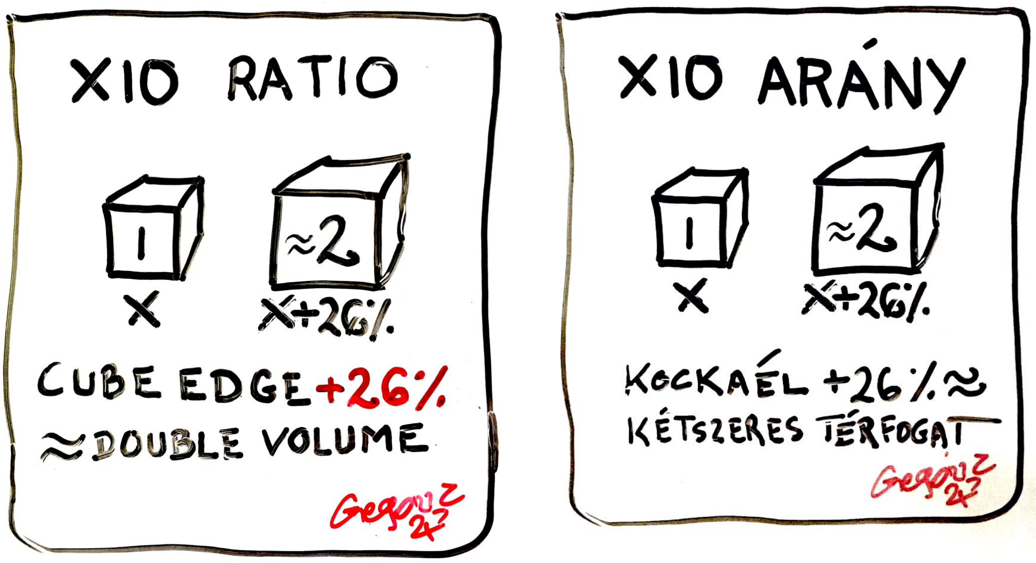

The XIO ratio as first drawn, in English and Hungarian. Kockaél + 26% ≈ kétszeres térfogat.

Abstract. The cube root of 2 is 1.2599210, so the edge of a cube of double the volume is a little over 26% longer than the original. Rounded to 1.26 this gives a rule that can be done in the head and is accurate to two hundredths of one per cent — and it is not a fact about cubes. Volume scales as the cube of any length, so adding 26% to any measurement of any shape whatever, from a teapot to a galaxy, about doubles what it holds. I call the relationship the **XIO ratio**: X for the longer edge, I for the shorter, and O for the nought in 0.26.

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1 The ratio

Add 26% to the edge of a cube to just about double its volume.

Subtracting the edge of a unit cube from the edge of a cube of twice its volume gives an irrational number that rounds to twenty-six hundredths. That is the whole of it.

$$\text{edge of the unit cube} = 1 \quad \cdot \quad \text{volume} = 1^3 = 1$$

$$\text{edge of the cube of volume } 2 = \sqrt[3]{2}$$

$$\mathbf{\sqrt[3]{2} - 1 = 0.2599210\ldots \approx 0.26}$$

I name this the **XIO ratio**, where **X** stands for the longer edge of the double-volume cube, **I** for the edge of the smaller one, and **O** for the nought in 0.26 — a nought that turns out to have a claim of its own, as section 4 shows. From it follows the **XIO formula**:

$$\mathbf{a \cdot 1.26 \approx b} \quad \text{where} \quad 2a^3 \approx b^3$$

1.1 How close is it?

Closer than a rule of thumb has any right to be. The rounding loses six thousandths of one per cent on the multiplier, and because the error is cubed it still comes to less than two hundredths of one per cent on the volume.

	Value	Error
$\sqrt[3]{2}$, the exact multiplier	1.2599210499	—
1.26, the rule	1.26	+0.0063%
1.26^3, the volume it gives	2.000376	+0.019%
exact increase needed	25.9921%	—
to halve instead: × 0.79	0.7936508	−0.006% on $\sqrt[3]{½}$
three steps of +26%	8.0045	+0.056% on 8
ten steps of +26%	1025.93	+0.19% on 1024

Table 1. The accuracy of the rounding. Even compounded ten times — a thousandfold increase in volume — the rule is still within a fifth of one per cent.

There is a second figure worth having. Since area scales as the square of length, the same 26% costs only $1.26^2 = 1.5876$ in surface — about 59% more skin for twice the contents. Anyone buying sheet metal, glass, cardboard or paint has just been told something useful.

2 It is not a fact about cubes

I first put this as a rule for cubes, then for spheres and for the regular and semi-regular solids — the Platonic, Archimedean, Catalan and Johnson families — and for any solid with a cubic determinant in its volume formula. That was too cautious. The rule is universal, and the reason is one line:

volume scales as the cube of any length

Take any shape at all. Scale every dimension of it by a factor k and its volume multiplies by k^3, whatever the shape was. The cube root of 2 is therefore the answer for everything, not for cubes. Add 26% to any single measurement of a scaled copy — an edge, a radius, a height, a diameter, a wingspan, a length of coastline — and the copy holds about twice as much.

A teapot 26% taller holds two cups instead of one. A lake 26% wider, deeper and longer holds twice the water. A planet of double the volume has a radius 26% greater; so does a star, and so does the sphere a galaxy's halo occupies. The scale of the object is irrelevant to the arithmetic. Only the requirement of similarity matters: the copy must be the same shape, scaled in every direction at once.

2.1 Where the rule stops — and why that is interesting

Geometry scales; physics does not. This is the honest limit of the rule and it is worth stating, because it makes the 26% more surprising rather than less.

Volume grows as the cube of length while cross-sectional area — which is what carries weight — grows only as the square. An animal of twice the volume is 26% longer but weighs twice as much on bones only 59% thicker in section, which is why a mouse enlarged to the size of an elephant would collapse, and why an elephant is not a scaled-up mouse. The same square-cube law governs why large ships need disproportionate structure, why insects can be shaped as they are and mammals cannot, and why a doubled-volume water tank needs more than 26% more wall thickness.

So the XIO ratio answers the geometric question exactly and the engineering question only as a first estimate. That is precisely the distinction a designer needs to keep in view, and it is why the rule belongs in the head as a quantity-and-capacity check rather than as a structural one.

3 Why the number is 26 and not something else

Nothing about the ratio requires it. The 26 is a consequence of $\sqrt[3]{2}$, and $\sqrt[3]{2}$ knows nothing of the integer 26 — round the decimal to two places and that is simply where the digits fall. What is striking is that they fall on an integer with independent claims of its own, and it is those claims, not the ratio, that made me name the thing.

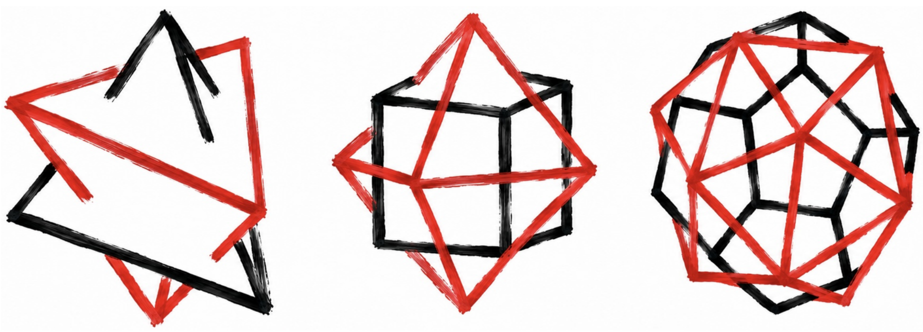
26 sits between a square and a cube. It is the only positive integer with a perfect power on either side of it: $25 = 5^2$ below, $27 = 3^3$ above. Extend the condition to all integers and zero joins it, sitting between $(-1)^3$ and 1^2. I treat that result at length in a companion paper, *Coefficial Divisibility*, where it is the subject rather than an aside; here it is enough to note that the nought and the twenty-six in 0.26 are both numbers with this property.

26 is how many times the cube's rotation axes meet its surface. Three four-fold axes through the face centres, four three-fold axes through the vertices, six two-fold axes through the edge midpoints — each crossing the surface twice:

$$3\cdot 2 + 4\cdot 2 + 6\cdot 2 = 6 + 8 + 12 = \mathbf{26}$$

Which is also, and not coincidentally, the count of the cube's characteristic elements: 6 faces, 8 vertices and 12 edges. That correspondence is the starting point of another companion paper, *The English Cube and Metatron Numbers*.

26 is the atomic number of iron, and the English alphabet has 26 letters. The iron deserves a section of its own, and has one below.



Tetrahedron – Tetrahedron   Cube – Octahedron   Dodecahedron – Icosahedron

3 × 2 = 5

Figure 1. The five Platonic solids as three dual pairs. Join the centres of a solid's faces and you get its dual: the cube gives the octahedron, the dodecahedron the icosahedron, and the tetrahedron another tetrahedron. Three pairs of two — and five solids, because the tetrahedron is its own dual and fills both places in its pair by itself. Arithmetic that looks wrong and is not, which is rather the business of this paper.

4 The iron

Of all the twenty-sixes, this is the one that has occupied me longest, and it is not in any of my other papers. Iron is element number 26: twenty-six protons in the nucleus and twenty-six electrons around it. What follows is what that number does — in a tomb, in a crystal, in a star, and in the blood.

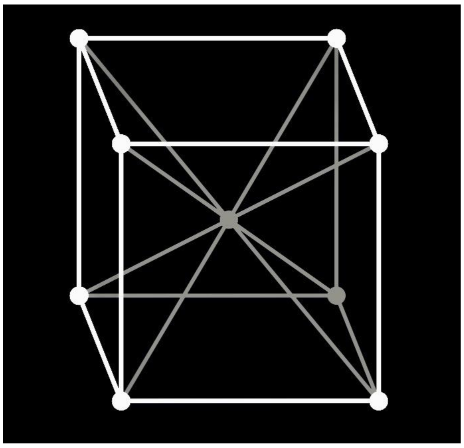
4.1 Beads from the sky

The oldest iron objects known are nine small tubular beads from two burials at Gerzeh in northern Egypt, securely dated to about **3200 BC**. They were strung into a necklace with lapis lazuli, gold and carnelian — which tells you what iron was worth then. And they were not mined. Analysis found cobalt and germanium at levels that occur only in meteoritic iron: the metal fell from the sky, and was hammered cold into sheets and rolled into tubes.

Working meteoric iron came first, at Gerzeh. **Smelting** — winning iron from ordinary ore — came much later, and the reason is a temperature: iron melts at 1538°C, far beyond any early furnace, while copper melts at 1085°C. That is the whole reason the Bronze Age precedes the Iron Age. The trick, when it came, was to avoid melting altogether — the bloomery furnace reduces the ore to a solid, spongy mass that is then hammered — and learning to hammer iron on the sky-metal at Gerzeh is what made that possible.

A footnote I cannot resist: the 2013 study that proved the beads meteoritic ran its neutron and X-ray analysis partly at the Wigner Research Centre for Physics in Budapest — the same institute whose cloud team ran the supercomputer search in my *Star Face*.

Once smelting was mastered, iron displaced bronze for a reason beyond hardness: it is everywhere, and it is not poisonous. Rome plumbed its cities in lead. An iron pot adds iron to the food cooked in it, and that is a benefit rather than a hazard — iron is an element the body requires. Only in overload does it harm, which is a different matter from a metal that is toxic at any dose.



Left: iron in two forms — a machined cube and rough pieces of the raw metal. Right: the body-centred cubic cell, iron's arrangement at room temperature: one atom at each corner of a cube and one more at its centre.

4.2 The atom, and a coincidence I am fond of

Iron has 26 protons and, when neutral, 26 electrons. Those electrons sit in four shells — successive layers around the nucleus, each holding only so many — and the count goes 2, 8, 14 and 2:

$$\mathbf{2 + 8 + 14 + 2 = 26}$$

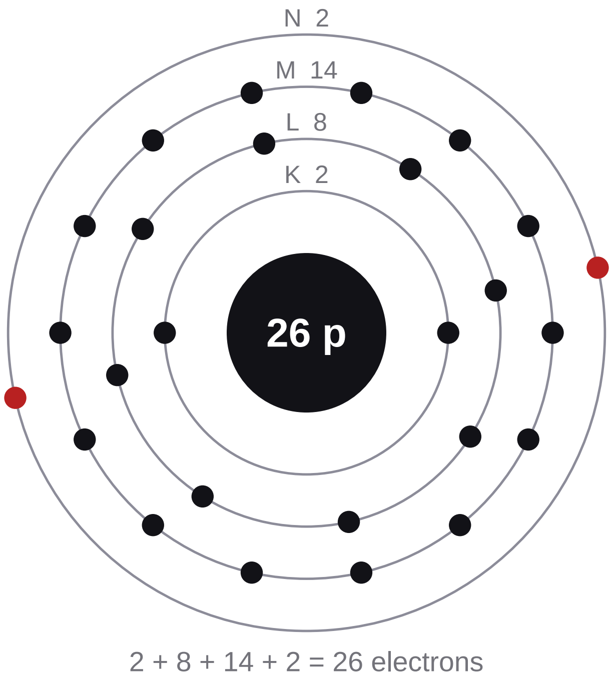


Figure 2. The iron atom. The dark centre is the nucleus, holding the 26 protons (p) that make the atom iron; alongside them sit neutrons, 30 of them in the commonest form, which do not affect which element it is. The four rings are the electron shells, labelled K, L, M and N from the inside out, holding 2, 8, 14 and 2 electrons. The two red dots on the outermost ring are the two electrons that do iron's chemistry — the ones it lends when it rusts, and when it carries oxygen in the blood.

It is the **26 protons** that make this atom iron. The neutrons vary — iron with 28, 30, 31 or 32 of them is all still iron — but change the proton count and the element changes with it: 25 is manganese, 27 is cobalt.

Now the size of it. The radius of an iron atom is **126 picometres**. A picometre, written pm, is a millionth of a millionth of a metre — so an iron atom is about 0.000000000126 of a metre across its radius, and you would need some four million of them side by side to span a millimetre. Measured in the older unit chemists still like, the ångström, which is a hundred picometres, that radius is **1.26 å**.

1.26 again. I want to be honest about what that is and is not. It is a coincidence, and it cannot be anything else: the metre was defined by people, the ångström is a human convenience, and an atom does not know either. Change the unit and the digits change. The cube root of two, by contrast, is 1.2599210 in every language and every system of measure there has ever been. So the two 1.26s are not connected, and I will not pretend otherwise. But a paper about adding 26% to an edge, whose author has spent years on the number 26, can be forgiven for noticing that the atom numbered 26 has a radius of 1.26 units in the scale chemists chose. I noticed. I am still smiling about it.

4.3 Iron is a cube

At room temperature iron's atoms arrange themselves in a **body-centred cubic** lattice: imagine a cube with an atom at each of its eight corners and one more sitting at the very centre. The cube's edge is 286.65 pm, a little over twice the atomic radius. Heat it past 912°C and the atoms rearrange into a **face-centred cubic** pattern — corners again, but now with an atom at the middle of each of the six faces instead of one in the middle of the solid. Past 1394°C it returns to body-centred, and at 1538°C it melts.

Both arrangements are cubes. The element that carries the number 26 builds itself, at every temperature it is solid at, on the shape whose 26 characteristic elements began this whole line of work. That is worth a moment's silence.

4.4 Where the stars stop

A star shines by fusion: light nuclei are pressed together into heavier ones, and the joining releases energy. Hydrogen becomes helium, helium becomes carbon, then oxygen, neon, magnesium, silicon. Each step is a smaller profit than the last. And at iron the profit runs out.

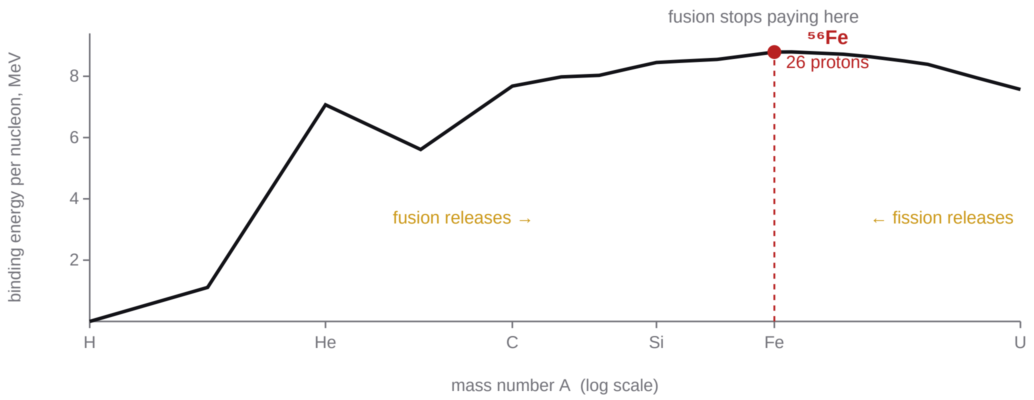


Figure 3. Why fusion stops at iron. The curve is the binding energy per nucleon — nucleon meaning a proton or a neutron, and binding energy meaning how tightly the nucleus holds together, measured in MeV, millions of electronvolts, the unit used for energies inside atoms. The higher the curve, the more tightly bound and the more stable the nucleus. Going uphill releases energy; going downhill costs it. Everything to the left of the peak can profit by fusing, everything to the right by splitting, and at the top — iron — neither pays. The mass number A on the bottom is simply how many nucleons the nucleus has — so iron-56 means 56 nucleons, being iron's **26 protons** plus 30 neutrons. The 56 names the isotope; the 26 names the element.

A detail worth stating precisely. Iron-56 is not quite the most tightly bound nucleus there is: **nickel-62** holds that record, at 8.7945 MeV per nucleon against iron-56's 8.79. What iron-56 does have is the lowest *mass* per nucleon of any nuclide at all. And nickel-62 cannot be reached in quantity by the route stars actually take, because that route — the alpha process — adds helium nuclei four nucleons at a time, and 62 is not a multiple of four.

And nickel's number is **28** protons, but the isotope in question is nickel-**62** — which is 26 read backwards. Idle, and I know it. But it comes with something that is not idle: **26 and 62 are the only two-digit integers whose halved digits add up to the difference of those digits**. Halve 2 and 6 to get 1 and 3; they sum to 4; and 6 − 2 = 4. The same holds for 62, and for nothing else in the range.

The halving is what does the work. A digit with no whole half cannot enter the question at all, so every number with an odd digit is excluded before it starts, and of the twenty two-digit numbers built from even digits alone, only 26 and its reversal satisfy the condition.

So the peak of the curve is held between the element numbered 26 and an isotope numbered 62, and no, I cannot make anything of it either.

So the chain runs to nickel-56, fourteen helium nuclei stacked, which is unstable and decays to cobalt-56 and then to **iron-56**, where it settles. Fusing anything further would consume energy instead of releasing it. The star's furnace goes out, the core has nothing left to hold itself up with, and it collapses — which is a supernova. The ash of that fire, scattered across space, is iron.

4.5 From the core of a star to the core of the blood

Everything after that follows from where the iron went. It is, by mass, the most abundant element on Earth. Being dense, most of it sank while the planet was forming, and it is down there still: the Earth's core is iron with some nickel, liquid on the outside and solid at the centre. That liquid iron moves, and moving iron makes a magnetic field — the field that deflects the solar wind and keeps this planet's atmosphere from being stripped away. Mars, whose core cooled, lost its field, and then lost most of its air.

And in your blood, at the centre of every haemoglobin molecule, sit iron atoms, each holding one oxygen molecule for the length of a heartbeat and letting it go where it is needed. Red blood is iron chemistry, using those same two outer electrons.

So the number 26 ends the life of a star, is flung out by its death, sinks to the middle of a planet, raises the shield that lets an atmosphere survive, and then carries the breath around the body of everything that breathes. That is a considerable career for an integer, and none of it is numerology — every step of it is in the textbooks.

4.6 And the letters

Now the part that makes scientists uneasy, and which I claim as an artist's liberty rather than as a finding. The numerical value of the Tetragrammaton — the four-letter name of God in the Hebrew Bible, read right to left as yod, hey, vav, hey, and valued by the ancient practice of gematria at 10 + 5 + 6 + 5 — is **26**. The English alphabet has 26 letters. The cube has 26 characteristic elements. Iron is element 26.

I do not claim a mechanism. There is none, and anyone offering one should be disbelieved. Gematria assigns values by a convention; the alphabet grew by accident; the cube's 26 is a theorem; iron's 26 is a fact of physics. Four unrelated systems, one number. A mathematician is right to call that coincidence, and I would not argue.

But it is the coincidence that set me going. Every paper I have written — the segments of the English Cube, the sandwiched integer, the sieve model, and this ratio — began with noticing that a number kept turning up where it had no business being, and asking what was actually true around it. The noticing was an artist's; what came of it had to survive arithmetic. I mention the Tetragrammaton because it is honest to say where the looking started, and because a reader who shares the instinct should know they are not the first to have it.

4.7 The words used here

Word	What it means
picometre (pm)	a millionth of a millionth of a metre. Four million iron atoms side by side make one millimetre
ångström (å)	a hundred picometres. Convenient for atoms: iron's radius is 1.26 of them
proton, neutron	the two kinds of particle in a nucleus. The proton count is what makes an element the element it is — 26 of them and it is iron
nucleon	a proton or a neutron, counted together
mass number (A)	how many nucleons a nucleus has. Iron-56 has 56
electron shell	a layer the electrons occupy around the nucleus, each holding only so many. Iron's four hold 2, 8, 14 and 2
binding energy per nucleon	how tightly the nucleus grips each of its parts. The higher, the more stable
MeV	million electronvolts, the unit for energies inside an atom. A very small amount of energy in everyday terms, and an enormous one per particle
fusion, fission	joining light nuclei, and splitting heavy ones. Both release energy, on opposite sides of iron
alpha process	the route stellar fusion takes, adding helium nuclei four nucleons at a time
body-centred cubic	atoms at the eight corners of a cube and one at its centre. Iron's arrangement when cool
face-centred cubic	corners again, with an atom in the middle of each of the six faces. Iron's arrangement when hot
smelting	winning metal from its ore. For iron it does not require melting — which is why it was possible at all, two thousand years after Gerzeh
gematria	an old practice of giving letters numerical values and adding them up

Table 3. Every term used above, in plain words.

5 The square, the cube, and the turning

There is a progression in the three facts above that I want to set out carefully, because it is easy to overstate.

25 is a fact about the plane: a square number, two dimensions multiplied. 27 is a fact about space: a cube, three dimensions. And the third property — the 26 crossings of the rotation axes — is the one you cannot demonstrate without time passing. A square is square at an instant; a cube is cubic at an instant; but a rotational symmetry has to be *performed*. You must turn the cube to show that it comes back to itself, and turning takes time.

I will not claim that this makes the symmetry four-dimensional. It does not: the axes are ordinary lines in space, and the symmetry group is a set of transformations, not a set of events. But of the three, it is the only one that a still picture cannot carry — and if one wants a name for a property that needs duration to be exhibited, time is the honest place to look. That is what I mean by calling 26 **the Transdimensional Number**: it is reached from the plane, from space, and from motion, by three unrelated routes.

6 A note on computers

If you check the sandwiched-26 property with a calculator or a graphing program, be wary of the machine. Rearranged, the condition is

$$x^2 + 2 = y^3$$

and software will happily offer you solutions that are not solutions:

Offered	$x^2 + 2$	y^3	Off by
x = 27,000, y = 900	729,000,002	729,000,000	2
x = 216,000, y = 3,600	46,656,000,002	46,656,000,000	2
x = 12,500, y = 2,500	156,250,002	15,625,000,000	not close

Table 4. Floating-point near-misses. The first two are wrong by exactly 2 — the machine has rounded away the very quantity in question.

The true statement is a classical theorem, and it is worth citing rather than arguing from scratch: **Fermat** proved that $x^2 + 2 = y^3$ has exactly one solution in positive integers, x = 5 and y = 3 — which is to say 25 and 27, with 26 between them. The modern proof runs through unique factorisation in the ring $Z[\sqrt{-2}]$.

7 What it is good for

Computers give more precise figures than any rule of thumb, and no one should pretend otherwise. But a number you can apply in your head, in a workshop or a warehouse or a meeting, without reaching for anything, has a value of its own. Add a quarter and a bit; you have doubled the contents.

Question	Answer by XIO
A tank of twice the capacity — how much bigger?	every dimension +26%
A box of half the volume — how much smaller?	every dimension −21%
Twice the volume — how much more material?	about +59% of surface
Eight times the volume?	+26% three times over
A thousandfold?	+26% ten times over

Table 5. The rule in both directions. Useful in design, manufacturing and logistics — procurement quantities, transport and storage capacity, and the budgets that follow from them.

Related work

The sandwiched 26 is treated properly in *Coefficial Divisibility*, grego.hu/cd. The cube's 26 characteristic elements are the subject of *The English Cube and Metatron Numbers*, grego.hu/mn. A companion ratio in the cube is *The VIO Formula*, grego.hu/vio — named on the same pattern, and from which this one takes its X, I and O.

Grego, Mayday 2024.