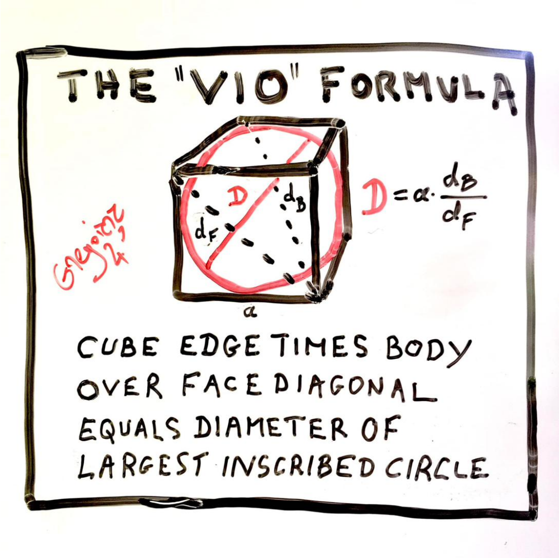
VIO Formula

Cube edge times body- over face-diagonal equals the diameter of the largest inscribed circle.

$$D = a \cdot \frac{d_b}{d_f}$$



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by **Grego** (Földvári Gergely)

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Abstract. The VIO formula is first of all a sentence. Cut a cube through the midpoints of six of its edges and the section is a regular hexagon, the largest that fits; the circle inscribed in it is the largest circle the cube will hold. Its diameter is the cube's edge multiplied by the body diagonal divided by the face diagonal — and it is longer than the edge itself. Two extensions follow: the same length is the median of the cube's largest inscribed triangle, and its square is the diagonal of the cube's largest inscribed square. No square roots are needed to state any of it — they appear only in the proof. A practical consequence: a ring is held by a box narrower than the ring itself.

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1 The name

I named the relationship which I seem to have discovered the **VIO formula**, in which **V** represents the two diagonals, **I** the edge, and **O** the largest inscribed circle of the cube. *Vio* in Latin means “I go” or “I travel”, from the verb *viare*; the noun is *via*.

In the original Hungarian wording: *Kocka él szorozva test-, per lapátló egyenlő legnagyobb beírt kör átmérő.*

“A loss in algebra can be a discovery in geometry.”

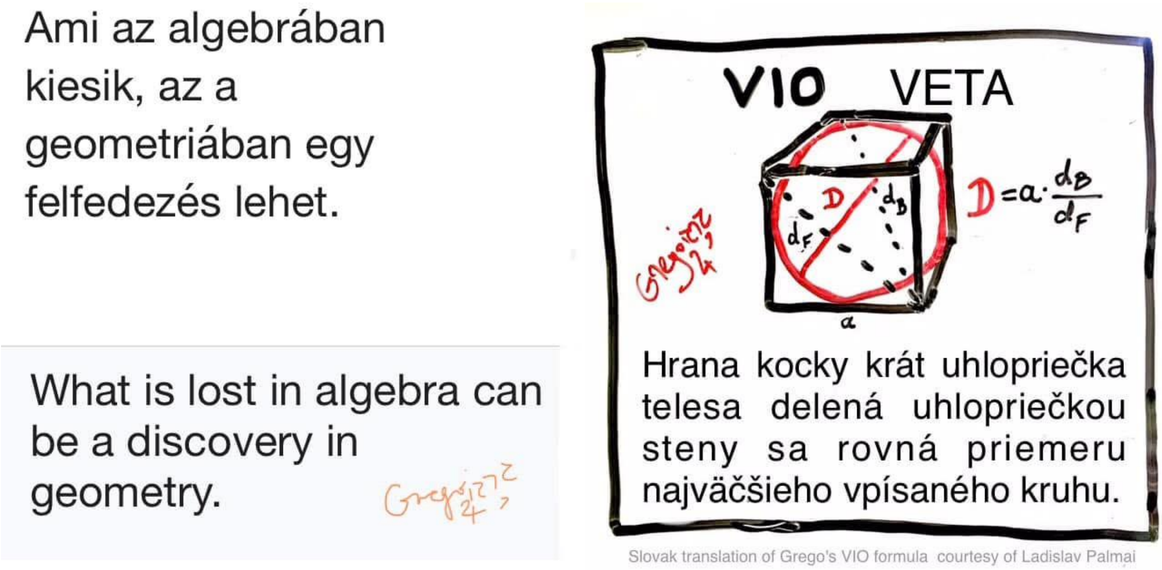

Figure 1. Left, the motto in Hungarian and English. Right, the Slovak wording, translated by Ladislav Palmai, 25 April 2024: *Hrana kocky krát uhlopriečka telesa delená uhlopriečkou steny sa rovná priemeru najväčšieho vpísaného kruhu.*

2 The proof

Among the infinitely many planes along which a cube can be cut in half, there are four that touch its surface at the midpoints of six edges. Each such plane is defined by the midpoints of two neighbouring edges together with the centre of the cube, and outlines a regular hexagon whose six vertices all sit at edge midpoints.

These regular hexagons are the largest that fit in a cube, since their entire perimeter lies on the cube's surface. By the same token the circles inscribed in them, touching the midpoints of their sides, are the largest circles the cube will hold: if the radius were any longer the circle would pass beyond the perimeter of the hexagon and so beyond the surface of the cube.

The radius x / 2 of a regular hexagon's incircle is the height of the equilateral triangle — six of them make the hexagon — whose sides are half the face diagonal of a unit cube. Pythagoras then gives, with x the diameter sought:

$$(x / 2)^2 + (\sqrt{2} / 4)^2 = (\sqrt{2} / 2)^2$$

which resolves to

$$\mathbf{x = \sqrt{3} / \sqrt{2}}$$

Many a learned algebra student will simplify that to $\sqrt{6} / 2$ at once — and in doing so lose the very thing worth seeing. Since $a\sqrt{3}$ is the body diagonal and $a\sqrt{2}$ the face diagonal of a cube of edge a, multiplying numerator and denominator by a leaves the value of x unchanged while rewriting it as

$$x = a\sqrt{3} / a\sqrt{2}$$

This can now be read as a fraction of the body diagonal over the face diagonal. If the edge changes by a factor a, the diameter changes linearly with it, since multiplying both sides by a preserves the equality:

$$a\,x = a \cdot (a\sqrt{3} / a\sqrt{2})$$

Which reads: the diameter of the cube's largest inscribed circle is always the edge multiplied by the body diagonal divided by the face diagonal. For a unit cube that is $\sqrt{6} / 2 \approx 1.2247449$ — longer than the edge, which is the part that surprises. Note that the statement itself never needed a square root: that is the whole point of not simplifying $\sqrt{3} / \sqrt{2}$.

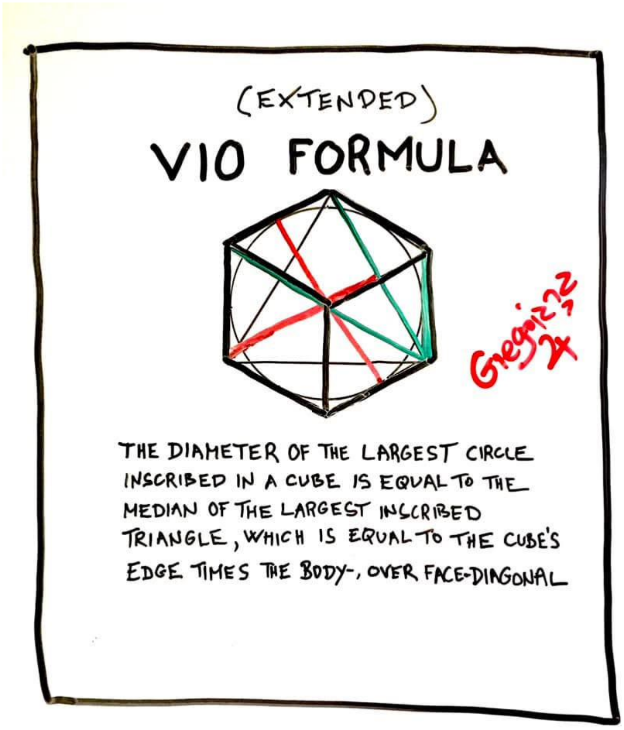

Figure 2. The hexagonal section with its incircle, drawn by hand. The diameter of the largest circle inscribed in a cube is equal to the median of the largest inscribed triangle, which is equal to the cube's edge times the body-, over face-diagonal.

3 The extension: the largest inscribed triangle

The largest triangle that fits in a cube is the equilateral one whose three vertices are corners of the cube, each pair joined by a face diagonal, so its side is $a\sqrt{2}$. The median of an equilateral triangle of side s is $s\sqrt{3} / 2$, which here gives $a\sqrt{2} \cdot \sqrt{3} / 2 = a\sqrt{6} / 2$ — exactly the diameter above. For a unit cube both are 1.2247449.

The diameter of the cube's largest inscribed circle is equal to the median of the cube's largest inscribed triangle, which is equal to the edge times the body-, over the face-diagonal.

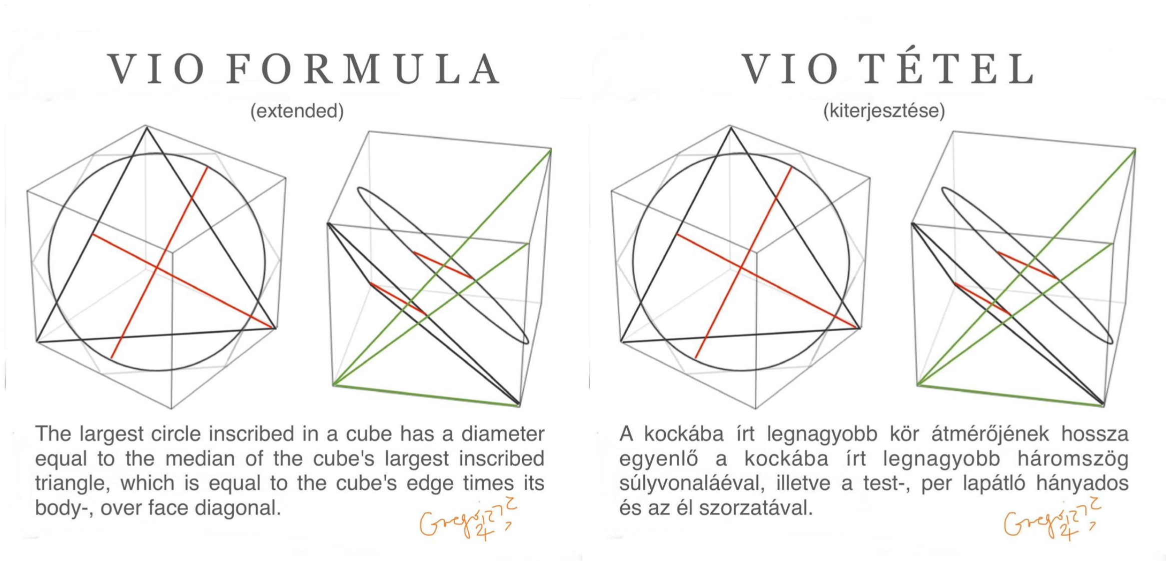

Figure 3. The extension, English and Hungarian. *A kockába írt legnagyobb kör átmérőjének hossza egyenlő a kockába írt legnagyobb háromszög súlyvonaláéval, illetve a test-, per lapátló hányados és az él szorzatával.*

4 The ultimate extension: the largest inscribed square

The largest square that lies entirely within a unit cube is not one of its faces. It is a tilted square of area 9 / 8, hence of side 3√2 / 4 ≈ 1.0606602 — the square at the heart of Prince Rupert's problem. Its diagonal is therefore exactly 3 / 2, and 3 / 2 is precisely the square of √6 / 2. So the third quantity closes the set:

The diagonal of the cube's largest inscribed square is equal to the square of the diameter of the cube's largest inscribed circle.

Writing D for that diameter, the three lengths of the cube's largest inscribed figures stand in a single chain:

Figure inscribed	Quantity	In terms of D	Unit cube
circle	diameter	$D = a \cdot d_b / d_f$	√6 / 2 = 1.2247449
triangle	median	D	√6 / 2 = 1.2247449
square	diagonal	D^2	3 / 2 = 1.5
square	side	D^2 / d_f	3√2 / 4 = 1.0606602

Table 1. The three largest figures a cube will hold, in terms of one length. d_b is the body diagonal, d_f the face diagonal, a the edge.

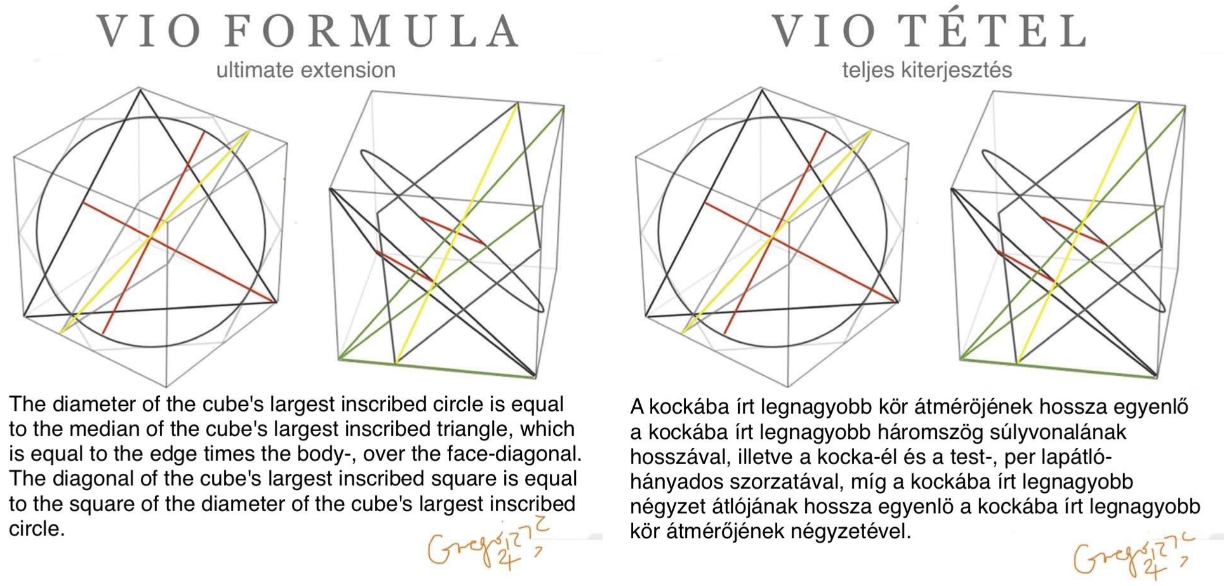

Figure 4. The ultimate extension, English and Hungarian. *… míg a kockába írt legnagyobb négyzet átlójának hossza egyenlő a kockába írt legnagyobb kör átmérőjének négyzetével.*

5 A wedding ring in a box, an optical cable in a lorry

The formula has a practical face, and it is the one worth remembering. A circle 22.47 per cent wider than the edge of a cubical box will still lie inside that box, because it can be tilted onto the hexagonal plane instead of laid flat against a face. Turned the other way round: to hold a ring of diameter d, a cubical box needs an edge of only $\sqrt{2} / \sqrt{3} \cdot d \approx 0.8165\, d$ — 18.35 per cent less than the ring is wide.

A wedding ring goes into a box narrower than the ring itself. A coil of optical cable rides in a crate whose side is shorter than the coil is across. For anyone packing, specifying or freighting round goods in square containers, the tilt is worth very nearly a fifth of the box — and the ratio that measures it is the body diagonal over the face diagonal.

The number quoted is the cube's. In an elongated container the tilt buys more still, since the coil can lean along the length; the cube is simply the case where the answer is exact and easy to say.

6 A meeting with the Metatron Numbers

The length $\sqrt{6} / 2$ is not a stranger. In the English Cube and its Metatron Numbers it appears as the fourth of the eight segment classes, the quarter-cube body diagonal, of which the cube holds 48. The largest circle a cube will contain has a diameter equal to that segment. The two results were arrived at separately and meet at this number.

7 Publication history

19 April 2024	First shared publicly by the author in the group *Polyèdres et particularités mathématiques*
19–21 April 2024	György Birkás: *Mateklecke* 4351 and 4359, the latter with proof
24 April 2024	*Bridges: Arts and Mathematics*
24 April 2024	*Number geometry*
25 April 2024	Slovak version, translated by Ladislav Palmai
12 June 2024	Extended by the author to the cube's largest inscribed triangle and square
11 May 2026	First on-line publication, Gondolkodás Öröme Alapítvány — mindenkijon.hu/p/mindenkijon-2026-13-nap

Acknowledgements

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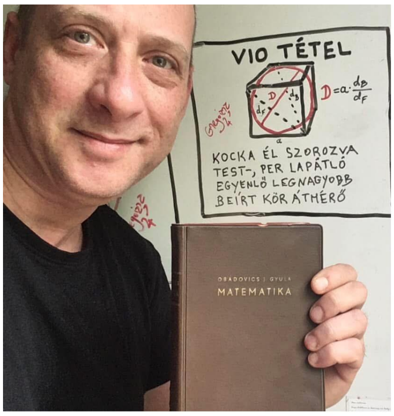
For the author's OEIS® contributions please visit: tinyurl.com/gregomat



Grego — Földvári Gergely, with Obádovics' *Matematika*.