The Tabakgasse Polyhedron

A Tabakgasse-poliéder

A tetradecahedron from the truncated cube, and the Eternal Flame of the Dohány Street Synagogue

by **Grego** (Földvári Gergely)

First published 13 August 2023 in Hungarian, 14 August 2023 in English. Bilingual edition, Budapest, 2 September 2026.

The lantern above the bimah, the construction shown on a paper-wrapped Rubik's cube, and the solid itself. 3D model courtesy of Vera Viana, PhD.

Abstract. Above the bimah of the Great Synagogue on Dohány Street in Budapest hangs an ornamented Eternal Flame — *ner tamid*, נר תמיד — whose lantern is a star polyhedron. Its core is a cube truncated in an unusual way: a tetradecahedron of 14 faces, 2 squares, 4 hexagons and 8 triangles, on which twelve pyramids are raised, with a David star set into each of the two squares in place of a pyramid. I name this solid the **Tabakgasse polyhedron**, after the name Theodor Herzl used for the synagogue built beside his birthplace. This paper gives its construction, its measured geometry and the reasons for thinking it has not been described before.

DOI 10.5281/zenodo.22261830 · grego.hu/tp · grego.hu/tp.pdf

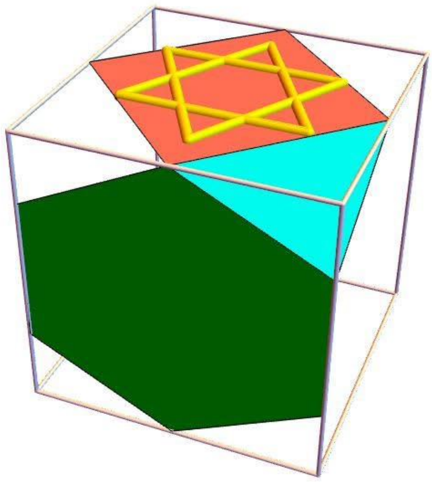
1 The lantern

Here is the spatial geometry of the magical Eternal Flame lantern above the *bimah* — the platform from which the Torah is read — of the Dohány Street Synagogue, shown on a Rubik's cube wrapped in a sheet of paper: a tetradecahedron of 14 faces, 2 squares, 4 hexagons and 8 triangles. It is a cube truncated at its corners and then extended with pyramids, and I hereby give it the name **Tabakgasse polyhedron** — because that is what Theodor Herzl called the synagogue built beside his birthplace, which today faces the square that bears his name; and because I have found no trace of such a solid anywhere, a judgement since supported by internationally recognised specialists whom I approached.

After the eight corners of the cube are cut away at the appropriate and identical angle, eight smaller pyramids (on triangular bases) and four larger ones (on hexagonal bases) are raised on a dozen of the solid's faces, while into the two squares a David star is set instead of a pyramid — one of them facing the assembled faithful of the *kehillah*, the community.

The key to building this singular, sacred and perhaps unique star-polyhedron lantern is the marking of the cube's faces and the cutting of its corners at the correct angle (compare *shechita kashrus*, kosher slaughter; the *tefillin*, the prayer cubes and straps whose every part is made from the hide of a kosher animal; and *brit milah*, ברית מילה, the circumcision of boys on the eighth day, according to the biblical commandment of entry into the Abrahamic covenant) — and then the raising, from the centroids of the triangular and hexagonal faces of the convex solid so obtained, of twelve pyramids in all, their apexes lying on lines perpendicular to those faces (compare *sefirot*, the divine emanations of the Kabbalah).

> *I ask anyone who knows of the existence of such a lantern, or of one structurally very similar, to be so kind as to send me information or a picture.*



Left: the synagogue on Dohány Street, photograph by the author. Right: the David star set into a square face, in place of a pyramid. Illustration by Sándor Kabai (1946–2024).

2 A lámpás — az eredeti magyar szöveg

Mutatom a Dohány utcai zsinagóga „bima” (pódium, ahol a Tórát olvassák) feletti, varázslatos örökmécses (נר תמיד / ner tamid / eternal flame) díszlámpásának térgeometriáját, egy papírlepelbe csomagolt Rubik kockán: tetradekaéder (14 lapú: 2 négyzet, 4 hatszög és 8 háromszög), egy rézsücsonkolt, majd piramisokkal kiterjesztett kocka, amelynek ezennel a „Tabakgasse-poliéder” nevet adom (mert Herzl Tivadar így hívta a szülőháza mellett épült zsinagógát, amely ma a nevét viselő térre néz és mert sehol nem találtam nyomát ilyen testnek, amit általam megkeresett, nemzetközileg elismert szakemberek véleménye is alátámasztott).

A kocka megfelelő és azonos szögben csonkított nyolc sarkának kimetszése után, a test egytucat lapjára nyolc kisebb (háromszög alapú), illetve négy nagyobb (hatszög alapú) piramist emeltek, a két négyzetbe pedig Dávid-csillag került piramis helyett, egyik néz a köhilá (közösség) jelenlévő hívői felé.

E különleges, szakrális és talán egyedülálló „csillagpoliéder”-lámpás megépítésének kulcsa a kocka lapjainak jelölése és sarkainak megfelelő szögben történő levágása (vö. „söhitá/shechita kásrusz”, azaz kóser mészárlás, illetve „tfilin”, azaz imakockák és szíjak, amelyek minden része kóser állatbőrből készül, valamint brisz mila, ברית מילה, fiúgyermekek körülmetélése a nyolcadik napon az ábrahámi szövetségbe-vétel bibliai parancsolata szerint), majd az így létrejött konvex test (14 lapú, tetradekaéder) három- és hatszög-lapjainak súlypontjaiból, a lapokra merőleges egyeneseken fekvő csúcspontokban találkozó, összesen 12 piramis emelése (vö. „szöfirójsz/szefirák/sefirot”, azaz I-ni emánáció a kabbala szerint).

> *Arra kérek mindenkit, hogy ha ilyen, vagy ehhez szerkezetileg nagyon hasonló lámpás létezéséről tud, akkor arról legyen kedves küldjön információt, képet nekem!*

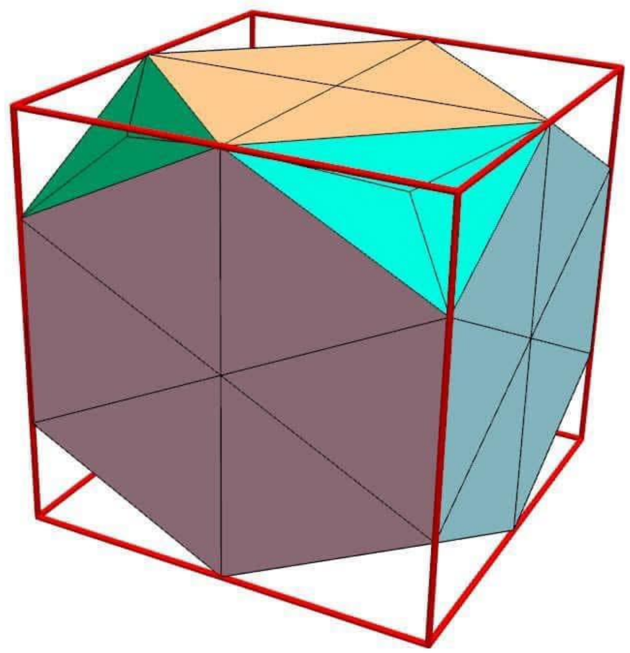
3 The construction

The construction of the Tabakgasse polyhedron and some of its basic geometric properties. Take a cube of unit edge.

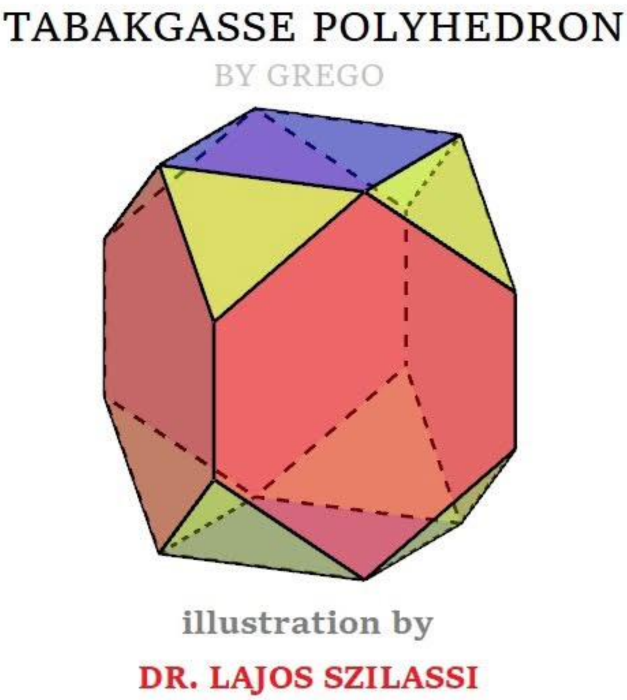
1. On two opposite faces of the cube — two faces sharing no edge — draw a square each, whose sides join the midpoints of the adjacent edges of those faces.

2. On the cube's other four faces inscribe hexagons that are not regular but are axially symmetric, each composed of six isosceles triangles, positioned so that the vertices of their two longer pairs of sides meet the vertices of the squares inscribed on the adjacent faces, while their shorter pairs of sides lie along the cube's edges and coincide with the equally shorter sides of the hexagons inscribed on the neighbouring faces.*

3. The cube is then truncated, starting from the sides of the squares, in the direction of the cube edge that ends at the corner to be cut off (a pyramid on a triangular base), along the longer sides of the hexagons lying on the two faces cut by the cutting plane.



The truncated solid within its cube, before the pyramids are raised. Illustration by Sándor Kabai (1946–2024).



The Tabakgasse polyhedron. Illustration by **Dr Lajos Szilassi**, discoverer of the Szilassi polyhedron.

3.1 A szerkesztés magyarul

1. Az egység élhosszúságú kocka két, közös éllel nem rendelkező (átellenes) lapjára egy-egy négyzetet rajzolunk, amelyek oldalai a lapok szomszédos élfelező pontjait kötik össze.

2. A kocka másik négy lapjára nem szabályos, de tengelyesen szimmetrikus, hat egyenlő szárú háromszögből álló hatszögeket írunk be úgy, hogy ezek két-két, hosszabbik oldalpárjának csúcsai a szomszédos lapokra beírt négyzetek csúcsaival érintkezzenek, a rövidebb oldalpárok pedig a kocka élein, a kocka szomszédos lapjaira beírt hatszögek szintén rövidebbik oldalaival fedjék egymást.*

3. A kockát ezután a négyzetoldalaktól kiindulva, a levágandó sarok (háromszög alapú gúla) csúcsában végződő kockaél irányában, a vágósík által metszett két lapon fekvő hatszögek hosszabbik oldalai mentén csonkítjuk.

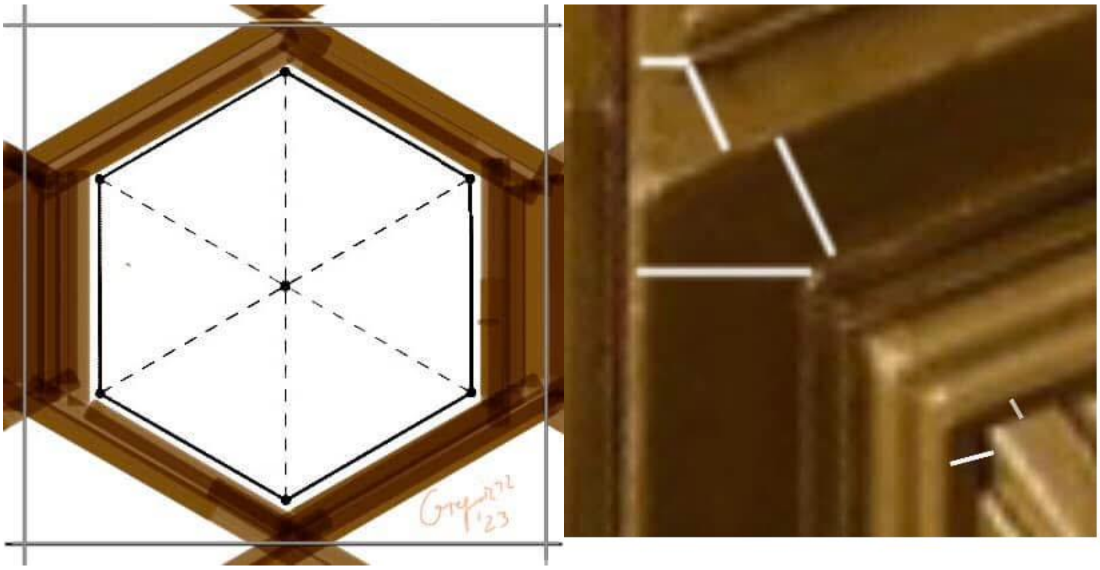
4 No ruler, no arithmetic

* The most remarkable aspect of the construction, to my mind, is that it needs no ruler and no calculation — only halving, cutting and, of course, logic. The square inscribed on two opposite faces of the cube is determined by the edge midpoints; but since it is impossible to create regular hexagons (composed of equilateral triangles) whose vertices and whose side midpoints alike coincide with the edge midpoints of the cube, we must use on the cube's other four faces the hexagons that are “the most regular” relative to the regular one: hexagons composed of isosceles triangles, whose base and height are both given, precisely determining the cutting plane.

* A Tabakgasse-poliéder megszerkesztésének számomra legkülönlegesebb aspektusa az, hogy nincs hozzá szükség vonalzóra vagy számolásra, csupán felezésre, vágásra és persze logikára: a kocka két átellenes lapjára beírt négyzetet az élfelező pontok határozzák meg, de mivel olyan (egyenlő oldalú háromszögekből álló) szabályos hatszögeket, amelyek csúcsai, illetve oldalfelező-pontjai egyaránt a kocka élfelező-pontjaival esnek egybe lehetetlen létrehozni, a szabályoshoz képest „leginkább szabályos” hatszögeket kell alkalmaznunk a kocka többi négy lapján, azaz egyenlő szárú háromszögekből álló hatszögeket, mely háromszögek alapja és magassága egyaránt adott, pontosan meghatározva a vágósíkot.

4.1 The regular hexagon in the lantern itself

A distinction worth stating plainly, since the two facts sit close together and can look contradictory. The hexagonal *face* of the polyhedron cannot be regular — that is what the paragraph above proves. But in the lantern as built, the framed wooden base beneath each hexagon lifts the pyramid clear of the truncated cube face, and on that raised frame the pyramid does stand on a genuinely **regular** hexagonal base. The impossibility is a property of the inscribed hexagon; the regularity is a property of the carpentry above it.



My quick sketch showing how the framed wooden base makes it possible to raise the pyramids on a **regular** hexagonal base after the truncation of the cube face. — Gyors vázlatom, amely megmutatja, hogy a keretes fatalapzat miként teszi lehetővé a piramisok szabályos (!) hatszögalapra emelését a kockalap csonkolása után.

5 The truncated solid, measured

After the truncation of the cube's corners in the manner described above, but before the pyramids are raised, the resulting polyhedron has the following properties. All figures are for a cube of unit edge. The construction fixes the truncation exactly: the four edges perpendicular to the two square faces are cut at one quarter of their length from each end.

	Count / Szám	Euler
vertices · csúcsok (C)	16	
edges · élek (É)	28	
faces · lapok (L)	14 — 2 squares, 4 hexagons, 8 triangles	
	C − É + L = 16 − 28 + 14 = 2	

Table 1. The convex tetradecahedron. — 1. táblázat. A konvex tetradekaéder.

5.1 The two square faces · a két négyzetlap

side · oldalhossz	$\sqrt{1/2}$	0.707107
area · terület	1/2	0.5
perimeter · kerület	$4\sqrt{1/2}$	2.828427
diagonals · átlók	1 (both)	1

5.2 The eight triangular faces · a nyolc háromszöglap

sides · oldalhosszak	√(1/2), √5/4, √5/4	0.707107, 0.559017
area · terület		0.153093
perimeter · kerület		1.825136
heights · magasságok	√3/4 and two equal	0.433013, 0.547723, 0.547723
angles · szögek		78.463°, 50.768°, 50.768°

5.3 The four hexagonal faces · a négy hatszöglap

sides · oldalhosszak	√5/4 (two pairs), 1/2 (one pair)	0.559017, 0.5
area · terület	3/4	0.75
perimeter · kerület	√5 + 1	3.236068
main diagonals · fő átlók	1, √5/2, √5/2	1, 1.118034, 1.118034

The three main diagonals are those joining opposite vertices through the centre. The hexagon's remaining six diagonals measure √13/4 ≈ 0.901388. — A három fő átló az átellenes csúcsokat kötő, középponton áthaladó átló.

5.4 A solid that houses a tetrakis hexahedron

The truncated solid is a hull for a tetrakis hexahedron: the latter can be housed inside the former. The relation is exact. Taking the tetrakis hexahedron in its canonical proportions — a cube bearing a pyramid of height one quarter of the edge on each of its six faces — the largest one that fits has a cube edge of exactly **two thirds** of the original cube's edge. At that size its six apexes reach the centres of the six original cube faces, so each apex touches the middle of one of the two squares or one of the four hexagons, while its eight cube corners sit at one sixth and five sixths along each axis, clear of all eight cutting planes. A tetrakis hexahedron of edge 0.68 no longer fits.

A csonkolt test burka egy tetrakisz hexaédernek: az utóbbi elhelyezhető az előbbin belül. A legnagyobb ilyen tetrakisz hexaéder kockájának éle pontosan kétharmada az eredeti kocka élének; hat csúcsa éppen az eredeti kocka hat lapközéppontját érinti.

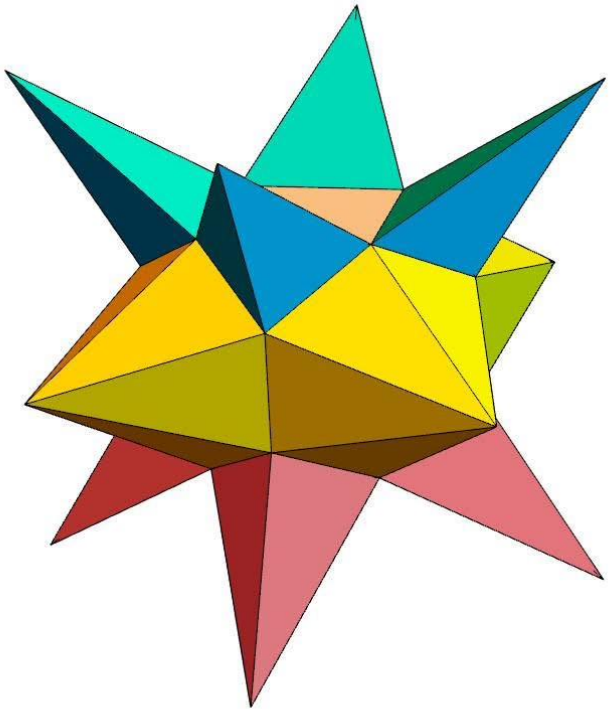
6 After the pyramids are raised

Two further stages follow. The lantern itself is the first of them: pyramids on the triangles and hexagons, with the two squares left for the David stars. The second is the complete augmentation, pyramids on every face.

Stage · Állapot	Vertices	Edges	Faces	Euler
truncated cube · csonkolt kocka	16	28	14	2
with 12 pyramids, as in the lantern · 12 piramissal	28	76	50	2
completely augmented · teljes kiterjesztéssel	30	84	56	2

Table 2. The three stages, each satisfying Euler's formula. — 2. táblázat. A három állapot, mindegyik teljesíti az Euler-összefüggést.

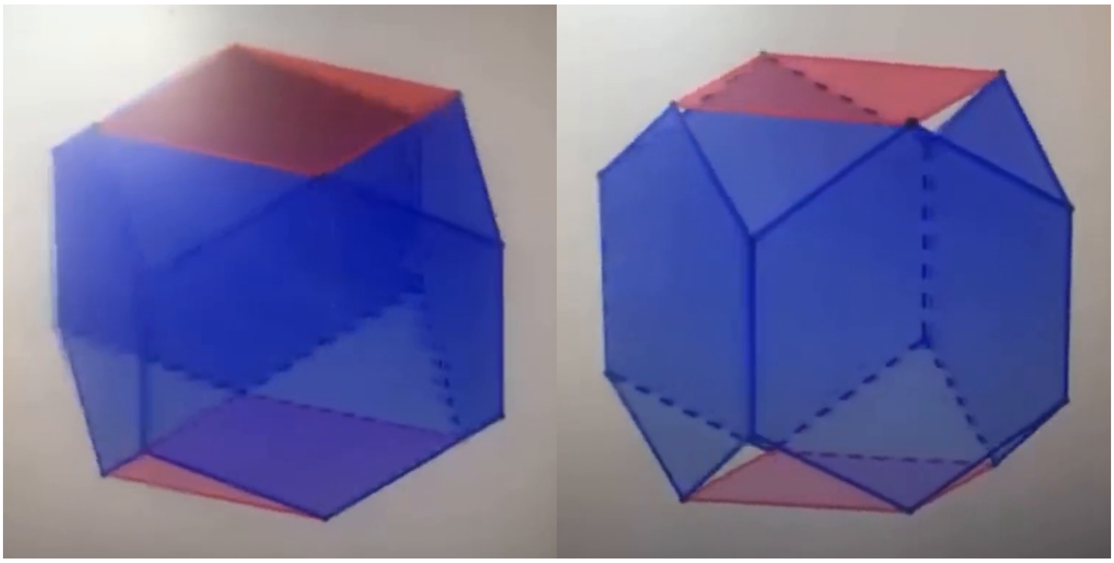
In the lantern, the twelve pyramids rise from the centroids of the eight triangles and four hexagons, their apexes on lines perpendicular to those faces — twelve, as the *sefirot* are counted in the Kabbalistic reading the lantern's makers may have had in mind.



The completely augmented Tabakgasse polyhedron: pyramids on all fourteen faces, 30 vertices, 84 edges, 56 faces. Illustration by Sándor Kabai (1946–2024).

7 The solid in rotation

I programmed and filmed two short animations. The first turns the cube-derived Tabakgasse polyhedron; the second turns the truncated-prism version, the one that admits regular hexagons and so corresponds to the lantern as actually built on its wooden frame. Both play at **grego.hu/tp**, and both files are attached to this PDF — open the attachments pane of your reader to save them.



Stills from the two animations. Left: the cube-derived solid. Right: the truncated-prism version with regular hexagons. Programmed and filmed by the author.

8 On the question of novelty

A word on what can and cannot be claimed. The Tabakgasse polyhedron is not a Johnson solid, and could not be: the Johnson enumeration admits only solids all of whose faces are regular polygons, while here the triangles are isosceles and the hexagons irregular. For the same reason it appears in no other standard catalogue, since those catalogues — Platonic, Archimedean, Catalan, Johnson, the uniform and star polyhedra, the deltahedra — enumerate only regular-faced or uniform solids. Its absence from them is therefore expected, and on its own proves nothing.

The evidence lies elsewhere. Searching the standard families and the literature turns up no convex solid with this face set — two squares, four hexagons, eight triangles, sixteen vertices, twenty-eight edges. The nearest named solids are the truncated cube (six octagons and eight triangles) and the truncated octahedron (eight hexagons and six squares), and neither is this one.

More telling than any search: three specialists in polyhedral geometry each made an illustration or model of this solid for me, and each reported never having encountered it before. **Vera Viana**, who made the 3D model; **Dr Lajos Szilassi**, discoverer of the Szilassi polyhedron, who made the drawing in section 3; and the late **Sándor Kabai** (1946–2024), whose illustrations appear throughout these pages, and some of whose last works are presented here. Three independent judgements, from people who would know, are what this claim rests on.

Három, a poliédergeometriában jártas szakember készített nekem illusztrációt vagy modellt erről a testről, és mindhárman úgy nyilatkoztak, hogy ilyen konvex testtel korábban nem találkoztak: Vera Viana, dr. Szilassi Lajos és a néhai Kabai Sándor (1946–2024).

9 Credits and colophon

Illustration	By
3D model of the solid (title page)	Vera Viana, PhD
The Tabakgasse polyhedron, line drawing (section 3)	Dr Lajos Szilassi
Truncated solid within its cube (section 3)	Sándor Kabai (1946–2024)
David star on the square face (section 1)	Sándor Kabai (1946–2024)
Completely augmented solid (section 6)	Sándor Kabai (1946–2024)
Lantern and synagogue photographs, sketches, animations	Grego

The Hungarian text was first published on 13 August 2023, the English summary on 14 August 2023. This bilingual edition adds the full English translation, corrects two numerical slips in the original list of measurements, states the tetrakis hexahedron relation exactly, and adds the note on the regular hexagonal base and the two animations.

Grego — Földvári Gergely, Budapest, 2 September 2026.