Music Calendar

The major scale and the Hebrew leap-year cycle

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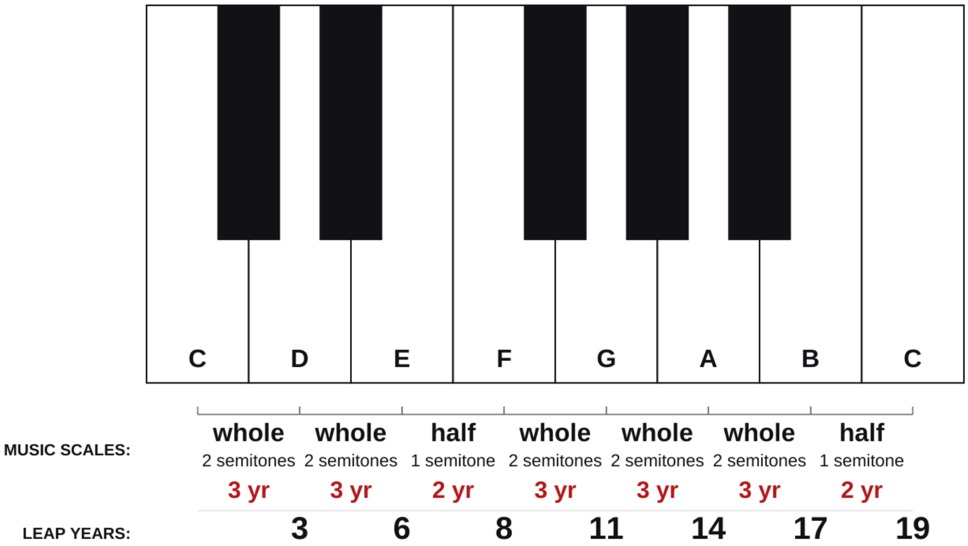
First published 19 February 2016. Typeset in Budapest, 4 September 2026. **Version 1.0.**

doi.org/10.5281/zenodo.22309690

Author's note. This paper looks at two systems with no historical connection — a scale and a calendar — and asks why they produce the same seven numbers. The noticing came first; the reason came afterwards.



One octave of the major scale, counted twice. Below each step, its size in semitones; below that, its size in years when the step is counted by notes rather than by intervals. The running total gives the leap years of the nineteen-year cycle.

Abstract. The Hebrew calendar inserts a thirteenth month in seven years out of every nineteen, at positions 3, 6, 8, 11, 14, 17 and 19. The major scale places seven notes among twelve semitones, in the pattern of whole and half steps every musician knows. Counting each step of the scale by the notes it spans — three for a whole step, two for a half — turns the scale into exactly that list of leap years. This paper sets out the correspondence, shows that it is not a coincidence but a theorem about distributing seven things evenly around a cycle, and identifies the infinite family it belongs to: the cycles of length 7k + 5, of which twelve is the chromatic octave, nineteen is the Metonic cycle, and twenty-six is the next. The seven modal scales turn out to be the seven ways of entering that cycle.

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1 Nineteen

I am a musician, and the number came to me through a song. Jimi Hendrix's *Little Wing* is built on a form of **nineteen bars** — an odd length, since almost all popular music divides into eight, twelve or sixteen. The harmony moves between G major and its relative minor, E minor, which are not two keys but one set of seven notes entered at two different points.

Nineteen is rare. I know of only two other places where it governs a structure of exactly this kind. One is the Hebrew calendar, which balances the moon against the sun over a nineteen-year cycle. The other is the major scale itself, once you count it in the right way. This paper is about why those two produce the same numbers.

2 The calendar

A lunar month is about 29.53 days and a solar year about 365.24, so twelve months fall eleven days short of a year. A purely lunar calendar drifts through the seasons; a purely solar one loses the moon. The Hebrew calendar keeps both by inserting a thirteenth month, Adar II, in certain years.

The rule is arithmetical and fixed. In each cycle of nineteen years, seven are leap years, and they fall at the same positions every time:

3 6 8 11 14 17 19

Nineteen years contain 235 lunar months, and 235 months and 19 years agree to within about two hours — the Metonic cycle, named for Meton of Athens but known earlier in Babylon. The Hebrew calendar has used it in its present codified form since the fourth century, traditionally attributed to Hillel II. The same cycle sets the date of Easter, which is why Passover and Easter drift together and apart.

The sequence of leap years is catalogued in the Online Encyclopedia of Integer Sequences as **A057349**, with the closed form $a(n) = \lfloor (19n + 5) / 7 \rfloor$.

3 The scale

The major scale takes seven notes from the twelve semitones of the octave. Between consecutive degrees there are whole steps and half steps, in the order

whole whole half whole whole whole half

which on a piano is the white keys from C to C. Measured in semitones these steps are 2, 2, 1, 2, 2, 2, 1, and they sum to twelve.

Now count differently. Instead of measuring the distance between two notes, count the notes themselves. A half step involves two notes, the one you leave and the one you reach. A whole step involves three, because a note lies between them. On this reckoning the steps are 3, 3, 2, 3, 3, 3, 2 — and they sum to **nineteen**.

The running total is the whole of it:

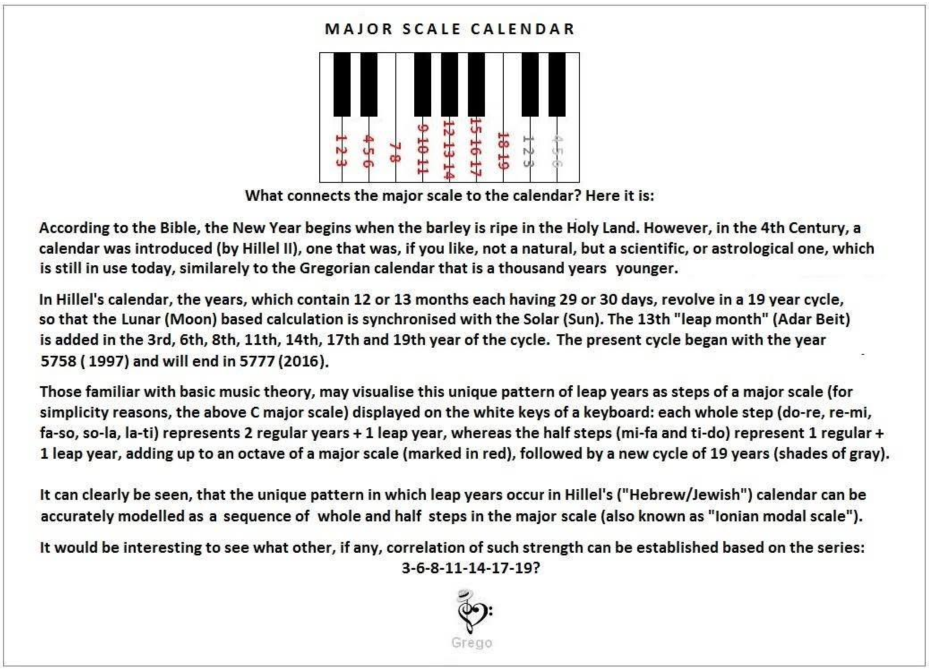
	1	2	3	4	5	6	7
step	whole	whole	half	whole	whole	whole	half
semitones	2	2	1	2	2	2	1
notes, or years	3	3	2	3	3	3	2
running total	**3**	**6**	**8**	**11**	**14**	**17**	**19**

Table 1. The major scale counted by notes. The bottom row is the leap-year list of section 2, arrived at without mentioning the calendar.

A whole step is two ordinary years followed by a leap year; a half step is one ordinary year followed by a leap year. Play the scale and you are counting out the cycle.

4 How it was first put

I posted this in February 2016, on social media, with the keyboard drawn out and the years written along the keys. The sheet is reproduced here as it stood, with one correction: the cycle current at the time ran from 5758 to 5776, not to 5777. Year 5777 is the first of the next cycle. The present cycle began with 5777 and ends with 5795; the Hebrew year 5786, which covers most of 2026, stands at position ten and is an ordinary year.



MAJOR SCALE CALENDAR

1 2 3 | 4 5 6 | 7 8 | 9 10 11 | 12 13 14 | 15 16 17 | 18 19 | 1 2 3 | 4 5 6

What connects the major scale to the calendar? Here it is:

According to the Bible, the New Year begins when the barley is ripe in the Holy Land. However, in the 4th Century, a calendar was introduced (by Hillel II), one that was, if you like, not a natural, but a scientific, or astrological one, which is still in use today, similarely to the Gregorian calendar that is a thousand years younger.

In Hillel's calendar, the years, which contain 12 or 13 months each having 29 or 30 days, revolve in a 19 year cycle, so that the Lunar (Moon) based calculation is synchronised with the Solar (Sun). The 13th "leap month" (Adar Beit) is added in the 3rd, 6th, 8th, 11th, 14th, 17th and 19th year of the cycle. The present cycle began with the year 5758 (1997) and will end in 5777 (2016).

Those familiar with basic music theory, may visualise this unique pattern of leap years as steps of a major scale (for simplicity reasons, the above C major scale) displayed on the white keys of a keyboard: each whole step (do-re, re-mi, fa-so, so-la, la-ti) represents 2 regular years + 1 leap year, whereas the half steps (mi-fa and ti-do) represent 1 regular + 1 leap year, adding up to an octave of a major scale (marked in red), followed by a new cycle of 19 years (shades of gray).

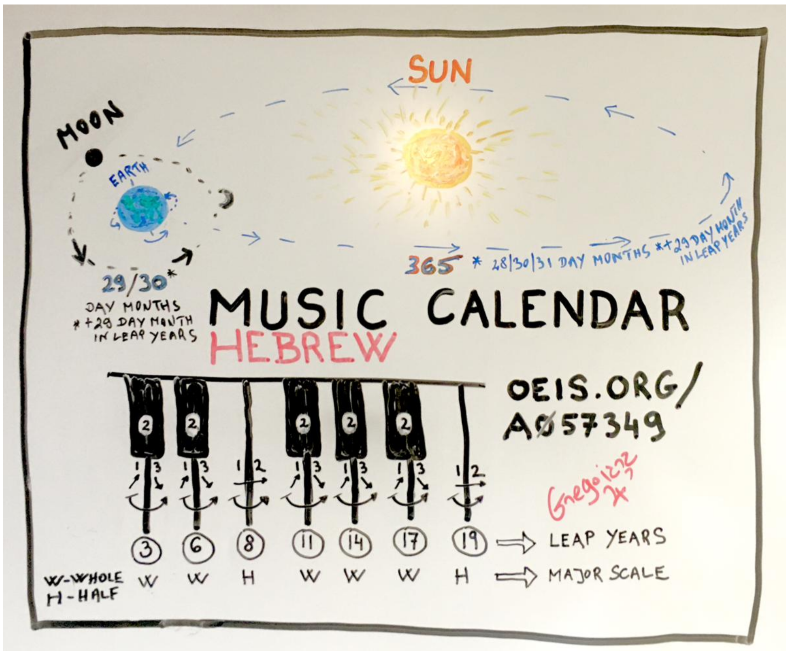
It can clearly be seen, that the unique pattern in which leap years occur in Hillel's ("Hebrew/Jewish") calendar can be accurately modelled as a sequence of whole and half steps in the major scale (also known as "Ionian modal scale").

It would be interesting to see what other, if any, correlation of such strength can be established based on the series: 3-6-8-11-14-17-19?

Grego

The original post, 19 February 2016. The red numbering runs through one nineteen-year cycle across the white keys; the grey numbering begins the next.

Eight years later the observation was submitted to the Online Encyclopedia of Integer Sequences and accepted as a comment on A057349 on 28 July 2024, with the illustration below linked from the entry.



The author's drawing. The moon governs the months, the sun the years, and the keyboard reconciles them: 2 for a half step, 3 for a whole step, nineteen in all.

5 Modal scales

The seven notes of the major scale can be entered from any of its seven degrees. Start on the second and the same notes give a different order of steps; start on the sixth and you have the natural minor. These are the **modes**, and they are not seven scales but one scale seen from seven positions.

Their names — Ionian, Dorian, Phrygian, Lydian, Mixolydian, Aeolian, Locrian — are Greek, but the usage is not. Ancient Greek theory applied those words to something else; the modern system was fixed by Glarean in his *Dodecachordon* of 1547, and the mismatch has been confusing students ever since. It is worth saying plainly, because the names suggest an antiquity the categories do not have.

Each mode is a rotation, and each rotation is a legitimate way of entering the nineteen-year cycle — which is to say, a choice about which year you call the first.

Mode		Semitone steps	In the cycle	Leap years
Ionian	major	2 2 1 2 2 2 1	3 3 2 3 3 3 2	**3 6 8 11 14 17 19**
Dorian		2 1 2 2 2 1 2	3 2 3 3 3 2 3	3 5 8 11 14 16 19
Phrygian		1 2 2 2 1 2 2	2 3 3 3 2 3 3	2 5 8 11 13 16 19
Lydian		2 2 2 1 2 2 1	3 3 3 2 3 3 2	3 6 9 11 14 17 19
Mixolydian		2 2 1 2 2 1 2	3 3 2 3 3 2 3	3 6 8 11 14 16 19
Aeolian	natural minor	2 1 2 2 1 2 2	3 2 3 3 2 3 3	3 5 8 11 13 16 19
Locrian		1 2 2 1 2 2 2	2 3 3 2 3 3 3	2 5 8 10 13 16 19

Table 2. The seven modes, each as a rotation of one pattern, and each as a rotation of the calendar cycle. Every row of the last column sums to nineteen. The Ionian row is A057349.

The modes are not a curiosity of theory. Dorian carries much of English, Irish and Balkan traditional music, and the modal jazz of the late 1950s made a whole idiom of staying on one rotation for bars at a time. Mixolydian is the sound of the blues and of most rock that leans on a flattened seventh. Aeolian is the default of pop and of film music in a sombre mood. Phrygian, with its half step at the bottom, is the flamenco cadence. Lydian was raised to a system by George Russell in 1953, whose *Lydian Chromatic Concept of Tonal Organization* argued that Lydian, not Ionian, is the true centre of tonality; the book shaped a generation of improvisers.

Locrian is the exception that proves the rule, and the reason is not the arrangement of its steps but where its fifth degree falls. In every other mode the fifth sits seven semitones above the tonic. In Locrian it sits at six, so the home chord is diminished — the only rotation of the seven with that defect.

Six semitones is the **tritone**, three whole steps, which divides the octave exactly in half and so belongs to no key in particular. It is the interval long known as *diabolus in musica*, the devil in music. Renaissance counterpoint forbade it under the teaching rule *mi contra fa est diabolus in musica* — not by any decree of the Church, as the story is usually told, but because the interval will not settle and leaves the ear without footing. A mode whose tonic and fifth stand a tritone apart has nowhere stable to rest. That, and not its step pattern, is why Locrian is barely used.

In the chord-scale theory taught since the 1970s, a chord symbol is read as an instruction about which rotation to play over it. A minor seventh chord invites Dorian, a dominant seventh invites Mixolydian, a major seventh with a raised fourth invites Lydian. An improviser choosing a mode is choosing where in the cycle to stand.

A caution about reach. The seven-in-twelve pattern is widespread — the medieval church modes, the maqam families Ajam and Nahawand, the Jewish prayer modes, the Indian thaat system in part — but it is not universal, and it would be false to claim otherwise. Indonesian *slendro* and *pelog* divide the octave quite differently, much Arabic practice uses intervals no piano can play, and Indian theory recognises far finer divisions. What recurs across cultures is not this scale but the principle behind it, which is the subject of the next section.

6 Why it happens

A correspondence this exact asks for an explanation, and there is one. Both systems are solving the same problem: place seven things around a cycle as evenly as the cycle allows.

The calendar must fit seven leap years into nineteen. Nineteen does not divide by seven, so the gaps cannot all be equal; the best that can be done is gaps of three years and two years, distributed so that the two short gaps sit as far apart as possible. The scale must fit seven notes into twelve semitones. Twelve does not divide by seven either, so again the steps come in two sizes, two semitones and one, with the two short steps as far apart as possible.

This is **maximal evenness**, named and studied by John Clough and Jack Douthett in 1991, and it is the same arithmetic that draws a straight line on a pixel grid. The answer it gives depends on only two things: how many items, and what remains when they are taken out of the cycle.

Seven into twelve leaves five. Seven into nineteen leaves five as well.

So both cycles need five long steps and two short ones, and both place them in the same order — short, long, long, short, long, long, long. Only the size of a step differs: one and two semitones, or two and three years. Counting by notes adds one to every step: a two-semitone step becomes three years, a one-semitone step becomes two. That is the whole mechanism, and it makes the correspondence a theorem rather than a coincidence.

7 The family

If what matters is the remainder, then any cycle leaving five when seven is taken out will produce the same pattern. Those cycles are the numbers 7k + 5, catalogued as **A017041**: 5, 12, 19, 26, 33, 40, 47 and onwards. Five is too small to hold seven positions, so the family begins in earnest at twelve.

Cycle	Short / long step	The seven positions	What it is
12	1 / 2	2 4 5 7 9 11 12	the chromatic octave
19	2 / 3	3 6 8 11 14 17 19	the Metonic cycle
26	3 / 4	4 8 11 15 19 23 26	the octave played up and back
33	4 / 5	5 10 14 19 24 29 33	—
40	5 / 6	6 12 17 23 29 35 40	—

Table 3. The family of cycles that give the major-scale pattern. Only the step size changes; the shape never does.

These are not abstractions. They are the equal temperaments in which the diatonic scale keeps its shape with the long and short steps differing by exactly one unit. Twelve-tone equal temperament is the tuning of the piano. **Nineteen-tone equal temperament is a real historical tuning**, proposed by Francisco de Salinas in 1577, used by Guillaume Costeley, advocated by Joseph Yasser in 1932 and composed in by Easley Blackwood. Twenty-six-tone temperament is used in microtonal practice today.

The third member deserves a word of its own. Counting by notes, as section 3 does, an octave of the chromatic scale is thirteen notes — twelve steps, but thirteen keys struck, since the octave itself must be counted. Play the chromatic scale up and then back down and you sound **13 + 13 = 26** notes. Twenty-six is where the family goes next, and it is also the plainest thing a musician can do with an octave.

Which yields the sharpest form of the result. In nineteen-tone equal temperament the major scale has steps of 3, 3, 2, 3, 3, 3, 2 and its degrees fall at 3, 6, 8, 11, 14, 17, 19. That is not an analogy to the leap years. It is the same list of integers.

The Hebrew leap-year cycle is the major scale in nineteen-tone equal temperament.

Each member of the family has a fifth slightly flat of the just ratio 3:2 — the direction that defines meantone temperament, of which twelve, nineteen and twenty-six are all members:

Temperament	Perfect fifth	In cents	Error against 3:2
12-tone	7 / 12	700.00	-1.96
19-tone	11 / 19	694.74	-7.22
26-tone	15 / 26	692.31	-9.65
33-tone	19 / 33	690.91	-11.05

Table 4. The fifth in each member of the family. Just intonation gives 701.96 cents.

8 What is claimed, and what is not

Claimed. The leap-year positions of the Hebrew calendar and the degrees of the major scale, counted by notes, are the same seven numbers. This is a theorem: both are the maximally even placement of seven items in a cycle that leaves remainder five, and there is only one such placement up to rotation. The seven modes correspond to the seven rotations of the cycle. The family of such cycles is 7k + 5, and its members are equal temperaments with a fifth flat of the just ratio, of which the nineteen-tone case makes the correspondence an identity rather than a resemblance.

Not claimed. That the framers of the calendar knew any music theory, or that musicians knew any calendar. There is no evidence of influence in either direction and I suggest none. Both traditions arrived at the same arrangement because they faced the same arithmetic, centuries and cultures apart. Nor is it claimed that the diatonic scale is universal: many traditions divide the octave quite differently, as section 5 sets out.

What remains is the pleasure of the thing. Two systems built for wholly different purposes — one to keep a festival in its season, one to make music — reached the same seven numbers, and neither knew.

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The author's OEIS® contributions to date: A389262 · A387890 · A387889 · A332001 · **A057349** · A006355 · A014530 · A051596 · A007047 · A005478

Acknowledgements. To the editors of the Online Encyclopedia of Integer Sequences, who accepted the comment on A057349 that this paper grew from, and to Mitch Harris, who contributed the sequence itself in 2000.

Version 1.0. The observation was first published on 19 February 2016 and entered in the OEIS on 28 July 2024. Every figure here was recomputed before publication, and the calendar arithmetic was checked against the standard leap-year rule.

Grego — Gergely Földvári, Budapest, 4 September 2026.