Hectoquads

A new class of square dissections, with a smallest identified

by **Grego** (Gergely Földvári)

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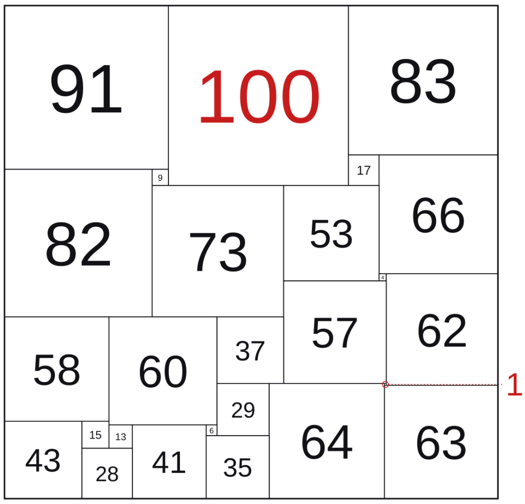
Author's note. This paper looks at square dissections and asks what is behind them — noticing the idiosyncrasy first, then establishing what is true around it. The noticing gives the classification; the counting gives the paper.



Left: the author's painting of the order-21 square of Duijvestijn, the smallest perfect squared square there is. Right: **26:274A**, the smallest hectoquad, reconstructed here from its Bouwkamp code.

Abstract. Medical accuracy, prevalence and incidence are almost always reported as whole percentage points, which is to say as integers from 1 to 100. Coupled with the intent to create sieve models for further use, that observation suggested the study of a new class of square dissections which would offer easily visualisable one-to-one correspondences to all their available percentage-point distributions composed of measured data involving populations equal to the orders in searchable dissection catalogues: those whose element sides all lie in that range and which hold both extremes, a square of side 1 and a square of side 100. I name them **hectoquads**. The perfect members form a finite universe — order at most 100, side at most 581 — so the class can be counted rather than merely sampled. Reading every published catalogue gives a census of **4,866**, of which exactly **eight** are perfect squared squares; the smallest of those is **26:274A**, order 26, side 274, and the smallest of any kind is the order-11 rectangle 11:185×168A. No hectoquad square in the census averages fifty, and exactly one has a median that does. This paper defines the class, gives 26:274A with its code and a full verification, sets out the census and the higher classes above it, and settles the problem the whole thing began with: a puzzle asking whether a mathematics teacher earned a pay raise, which turns on whether a class's scores can tile a square and on which statistic summarises them. Four clauses of its wording cut 244,099 dissections to one.

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1 The problem this began with

Everything here grew out of a puzzle I posted in English and Hungarian on 14 June 2022, and which was later set nationwide by the Joy of Thinking Foundation — Gondolkodás Öröme Alapítvány, backed by the Alfréd Rényi Institute of Mathematics — in their open online problem-solving project *minden·ki·jön*.

Should the maths teacher get a pay raise?

> *Everyone in the classes of a maths teacher happened to perform differently on their final tests; in each class, one pupil unfortunately achieved only the minimum (one percentage point), but luckily there was also one student in each class who got the highest possible score (hundred percentage points).*
>
> *The next day, students in his most junior and most senior classes were asked by the teacher to completely fill the two squares he had drawn with chalk on the school yard's concrete with smaller ones, but without overlaps or gaps in between and in such a way that their numbers would equal the student count of the classes and the lengths of their sides would correspond exactly to the respective percentage-point integers achieved on their tests. Students were allowed to use any means of help (calculator, computer, smart phone, library, internet, etc.) and eventually completed the task with success.*
>
> *In the mean time, looking out the windows of his office, the School Director watched the busy students in awe before angrily summoning and questioning the maths teacher who explained to the Director what was precisely taking place outside. The School Director then proclaimed that he'd only approve of next year's pay raise for the maths teacher if the students of at least one of the two classes managed to solve the task on the school ground and if at least one of the two classes had not failed the test as a whole, or in other words, they had achieved at least fifty percentage points in their statistically weighed overall score.*
>
> ***Question: why did or why didn't the maths teacher get a pay raise?***

1.1 Az eredeti magyar szöveg

Jár-e fizetésemelés a matektanárnak? Egy matematikatanár osztályaiban véletlenül minden osztálytárs más-más eredményt ért el az évvégi teszteken, ráadásul úgy, hogy osztályonként egy-egy tanuló munkája sajnos csak minimálisan (egy százalékpont) volt értékelhető, noha szerencsére mindegyik osztályban akadt egy olyan tanuló is, aki a lehető legjobban (száz százalékpont) teljesített.

Másnap, a legalacsonyabb és a legmagasabb évfolyamú két osztály diákjai azt a feladatot kapták a matektanártól, hogy az iskolaudvar betonjára általa krétával felrajzolt két négyzetet teljesen kitöltve, azokat takarás- és hézagmentesen úgy osszák fel kisebbekre, hogy ezek számossága adja ki az osztálylétszámukat, az oldalhosszak pedig egyezzenek meg a teszten elért, egészszámú százalékpontjaikkal. A diákok a feladat megoldásához bármilyen segédeszközt igénybe vehettek és végül sikerült is megoldaniuk azt.

Az igazgató közölte, hogy csak akkor hajlandó fizetésemelést adni, ha legalább az egyik osztály diákjai sikeresen megoldják az udvari feladatot és a statisztikailag összesített eredmény szerint a kettőből legalább az egyik osztály nem bukott meg a teszten, tehát legalább ötven pontot elért. **Kérdés: miért kapott vagy miért nem kapott fizetésemelést a tanár?**

1.2 The original sieve model

This problem is where the sieve model began, and it is worth saying so plainly, because the three sieves that later govern *Star Face* are already at work here — applied not to numberings of a polyhedron but to the statistics of a class.

The Director asks for a single figure summarising a population. Three are available — mode, mean, median — and the problem puts each to a different sieve.

Unambiguity asks whether the figure exists at all, and names one thing when it does. The mode fails at once: every pupil scored differently, so no value recurs and there is nothing left for the mode to point at. The sentence *everyone happened to perform differently* is not colour. It is the first sieve, doing its work in the opening line.

Equilibrium asks whether the figure balances what it summarises. The mean answers yes — it is the point about which the scores weigh the same on either side.

Reliability asks whether the figure holds steady when the population runs to extremes. The median answers yes — it names the value that halves the class, and neither a lone 1 nor a lone 100 can pull it anywhere.

So two figures come through, not one, and they disagree about fifty. Which of them the Director's wording licenses is the question section 5 answers, and not in the way the arithmetic first suggests — the answer turns on **a property of the median that conventional reporting hides**. The same three words later sift 6,223,974 numberings of Escher's solid down to a single deepest fold. The model was built here first, on a school yard, with chalk.

The puzzle has two halves and they resist different tools. Whether the yard task can be done at all is a question of existence, and the permission to use a computer and a library is not scene-setting — it is a statement that the first half is settled by search, not at the desk. Whether a class passed is a question about a statistic, and it is settled by reading the word *statistically* properly. Section 5 answers both.

2 Hectoquads

A **squared square** is a square cut into smaller squares with no gaps and no overlaps. It is **perfect** if no two of those squares are the same size, and **simple** if no part of the dissection forms a smaller squared rectangle. The **order** is the number of pieces. The lowest order at which a simple perfect squared square exists is 21, proved by Duijvestijn in 1978; that square has side 112 and is unique to its order.

Diagnostic accuracy, test reliability, prevalence, incidence and most other clinical and demographic quantities are reported as rounded percentage points — whole numbers from 1 to 100. Carrying that constraint into the geometry gives a class of dissections I have named as follows.

2.1 Why one to a hundred, and why both ends

The bound is not arbitrary. A diagnostic test is described by its sensitivity and specificity, a population by its prevalence and incidence, a study by its confidence level — and in practice every one of these arrives as a whole percentage point. Nobody reports a specificity of 87.3164 per cent to a clinician; they report 87. The reporting convention quantises the whole field onto the integers 1 to 100.

My *Star Face* model reads medical diagnosis through a polyhedron, sieving numberings of Escher's solid by equilibrium, unambiguity and reliability until a bounded landscape remains. The squared square offers a second stage for the same idea and one that quantises naturally: an infinite universe of distinct dissections, reduced by a single constraint to something finite and countable, and reducing in turn to a singularity — a smallest member. Hectoquads are one such reduction.

Definition. A **hectoquad** is a squared square or squared rectangle in which every element side s satisfies $1 \le s \le 100$, and in which both extreme values are present: it contains an element of side 1 and an element of side 100. The class takes in simple perfect squared squares (SPSS), compound perfect squared squares (CPSS), simple imperfect squared squares (SISS), simple perfect squared rectangles (SPSR) and simple imperfect squared rectangles (SISR).

Both halves of that definition earn their place. The bound carries the percentage scale into the geometry; the two extremes carry the class of pupils, one of whom scored 1 and one of whom scored 100. Dissections that meet the bound without reaching both ends are not hectoquads, but they are not nothing either — they model narrower scales than the one the name refers to, and they are far commoner. Two names for them:

A **half hectoquad** reaches one extreme only — either a 1 or a 100, not both; an **inset hectoquad** reaches neither, sitting wholly inside the scale without touching either wall. Among the simple perfect squared squares of orders 21 to 37 there are 63 half and 29 inset against 8 hectoquads — one hundred dissections inside the scale in all. Both tiers are open to further study; this paper counts them and sets them aside.

2.2 How large a hectoquad square can be

The definition puts a hard ceiling on the squared squares of the class — not on the rectangles, whose long side is bounded only by their area. It is worth deriving. The elements are distinct integers no greater than 100, so the most area any hectoquad can possibly enclose is the sum of the squares of *all* of them, $1^2 + 2^2 + \ldots + 100^2$. By the standard formula

$$\sum_{k=1}^{n} k^2 = n(n+1)(2n+1) / 6$$

that total is 100 · 101 · 201 / 6 = **338,350**, and since the area of the containing square is its side squared, the side cannot exceed √338,350 = 581.68. Sides are whole numbers here, so:

Every hectoquad squared square has side at most 581.

The bound cannot in fact be reached: 338,350 is not itself a perfect square, and no dissection using all hundred elements is known in any case. The known hectoquad squares sit between 38 and 56 per cent of it — 26:274A uses 47 per cent of the available side, and the largest, 35:324A, reaches 56 — so the class is not merely finite in principle but small, and boxed into a range narrow enough to search.

The same argument bounds the order. A perfect dissection has no two elements alike and only a hundred integers are available, so no perfect hectoquad can have more than a hundred pieces. Side at most 581, order at most 100: the perfect part of the class is a closed and finite universe, which is what makes a census possible rather than a collection of examples. The imperfect members, whose elements may repeat, are not bounded this way.

3 The smallest hectoquad squared square: 26:274A

The smallest hectoquad has **order 26** and **side 274**. In the catalogue of order-26 simple perfect squared squares it is **26:274A**, found by A. J. W. Duijvestijn in 1993.

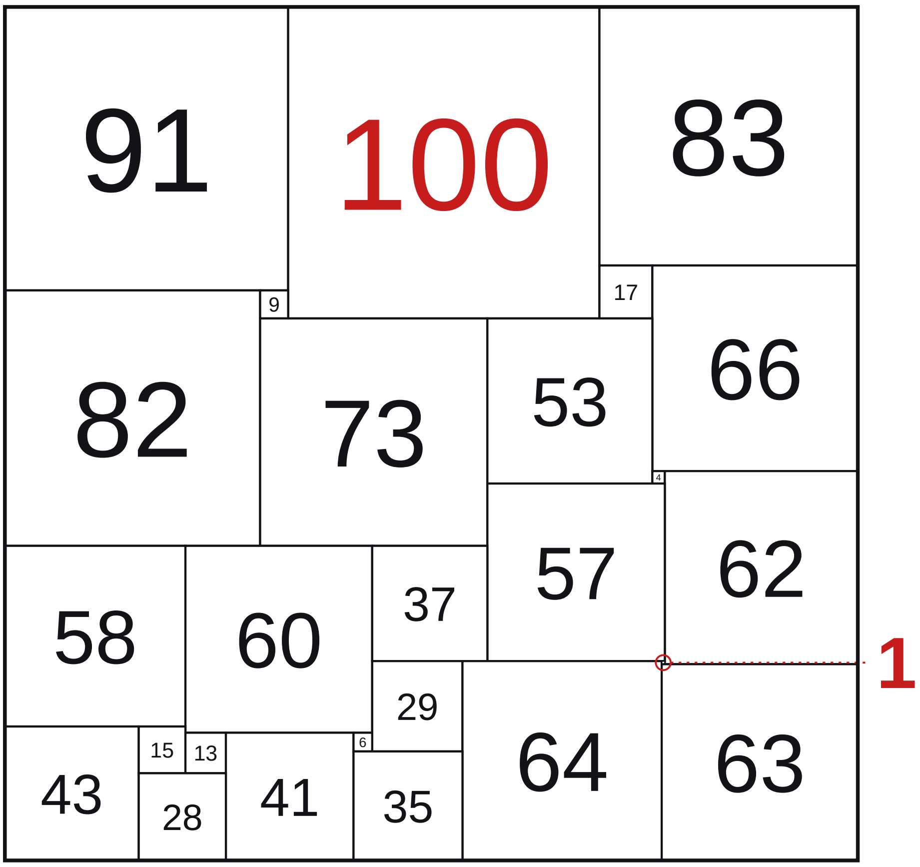


Figure 1. **26:274A**, reconstructed directly from its Bouwkamp code rather than copied. Twenty-six squares, every side different, the smallest 1 and the largest 100. The unit square is ringed in red; at this scale its side is barely a printer's hairline, which is the subject of section 6.

Its 26 element sides are

1, 4, 6, 9, 13, 15, 17, 28, 29, 35, 37, 41, 43, 53, 57, 58, 60, 62, 63, 64, 66, 73, 82, 83, 91, 100

and its Bouwkamp code, which records the dissection itself rather than merely its parts, is

```
26 274 274 (91,100,83) (17,66) (82,9) (73,53) (4,62) (57) (58,60,37) (29,64,1) (63) (43,15)
(13,41,6) (35) (28)
```

3.1 Verification

The code was decoded and the dissection reconstructed square by square on a 274 × 274 grid. All 75,076 cells are covered exactly once: no overlaps, no gaps. The 26 sides are distinct, the smallest is 1, the largest is 100, and the areas sum precisely to the whole:

$$\Sigma\, s^2 = 75{,}076 = 274^2$$

The unit square sits at (211, 210), away from every edge — as it must, since the smallest square in a perfect squared square can never touch the border.

4 The census

Every published catalogue of squared squares and squared rectangles was read and filtered: each dissection kept only if all its elements lie between 1 and 100 and both extremes are present. The results below are counts, not estimates. Where a category has been enumerated exhaustively the count is exact; where enumeration stops, the table says so.

Category	Orders	Hectoquad	Half	Inset	Summa	Smallest hectoquad
SPSS — simple perfect squared squares	21–37	**8**	63	29	100	26:274A
CPSS — compound perfect squared squares	24–39	**0**	1	0	1	—
SPSR — simple perfect squared rectangles	9–21	23	762	858	1,643	11:185×168A
SISS — simple imperfect squared squares	13–32	4,418	159,743	39,714	203,875	21:168
SISR — simple imperfect squared rectangles	9–22	417	22,642	15,421	38,480	15:184×160A
Total		**4,866**	**183,211**	**56,022**	**244,099**	**11:185×168A**

Table 1. The hectoquad census, with the half and inset tiers of section 2.1. Orders 21 to 37 are the complete enumerated range for simple perfect squared squares, so that count of eight is exact. **Summa** is the sum of the three tiers: every dissection lying inside the 1 to 100 scale, whether or not it reaches the extremes. The imperfect families dominate it — allowing an element value to repeat multiplies the field roughly two thousandfold. Average element sizes run from 33.16 to 45.77 among the eight, and from 2.41 to 63.64 across the whole summa. Two of the rectangle catalogues were truncated on the server; they were read from the intact bouwkampcode masters, and the rebuilt files later confirmed the counts exactly.

The perfect squared squares are the rarest of all, and by a long way. Orders 21 to 37 hold 27,940,214 simple perfect squared squares between them and exactly **eight** are hectoquads — fewer than one in three million. At order 26 it is one in 441[*]; by order 37 it is one in 17,086,918.

The half and inset tiers of section 2.1 are far larger. Across every family catalogued there are 183,211 half hectoquads and 56,022 inset ones, so of the 244,099 dissections lying inside the scale, the hectoquads proper are barely two per cent.

[*] 1 to 441 is precisely the worth scale of my *Limbo Match* game, studied in *The Mathematics of Grego's Limbo Match*, doi.org/10.5281/zenodo.22260942

Name	Order	Side	Mean	Median	Found by
26:274A	**26**	274	**45.7692**	**44–52**	A. J. W. Duijvestijn, 1993
29:221A	29	221	33.5517	29	Anderson, Johnson and Pegg, 2011
29:264A	29	264	38.2069	26	Anderson, Johnson and Pegg, 2011
30:246A	30	246	34.8667	27	J. D. Skinner
31:228B	31	228	33.1613	29	James Williams, 2013
33:279A	33	279	37.1818	27	James Williams, 2013
35:324A	35	324	45.1429	42	James Williams, 2016
37:317D	37	317	43.9730	38	James Williams, 2020

Table 2. The eight hectoquad simple perfect squared squares, smallest order first — the complete list over orders 21 to 37. The median is the value that halves the population, which for an even order is an interval rather than a number; section 5 sets out why that matters. The averages are shown because they are what a reader expects to see, and because the comparison between the two columns is the whole of section 5. Only 26:274A reaches fifty, and only on the median.

Two things in that table are worth pausing on. The smallest hectoquad, 26:274A, also carries the *highest* average of the eight — the first member found is also the best-scoring class. And not one of the eight averages fifty. Neither, as section 8 shows, does any other hectoquad square in the census.

4.1 The higher classes

Nothing obliges the scale to stop at a hundred. Admit every element side between k and $100k$, insisting as before that both extremes appear, and each value of k gives a class of its own — the second running 2 to 200, the third 3 to 300, and so on without end. No class is empty, since multiplying any hectoquad by k produces a member of the k-th, and each class has room for far more than those rescaled copies. The ladder is infinite.

Class	Elements	Side at most	Members	Smallest
first	1–100	581	**8**	26:274A, 1993
second	2–200	1,639	128	26:432A, 1993
third	3–300	3,007	335	29:558A
fourth	4–400	4,627	468	30:705M, 2013
fifth	5–500	6,464	358	28:848H

Table 3. The first five classes among simple perfect squared squares of orders 21 to 37. Side bounds follow the same sum of squares used in section 2.2.

The first class is the outlier: eight members against the second class's 128 and the fourth's 468. The higher classes are not rarer for being wider — they are commoner. Whatever makes the hundred scale so thin, it is particular to that scale, and the second class demonstrates it handsomely by opening at order 26 as well, with 26:432A, an even class whose median is again an interval rather than a number.

The ladder is infinite, and it is wide as well as tall. Each class has its own half and inset tiers, and its own imperfect members — the counts above are only the perfect squared squares that reach both extremes. What the census does for the first class could be done for any of them, and the figures would be far larger.

4.2 What the census could become

Each dissection here is a set of whole percentage points whose squares happen to sum to a perfect area. Read as data rather than as geometry, the census is a library of 244,099 distributions with populations from 9 to 39 — every one of them a class of pupils, a cohort of patients, a sample of respondents — of which 4,866 span the full scale from a 1 to a 100.

What makes them unusual as data is that each one draws itself. The dissection is not an illustration of the distribution; it *is* the distribution, laid out so that every value occupies area in proportion to its own square, with no gaps, no overlaps and no choices left to the person drawing it. A bar chart is a decision about how to show numbers. A hectoquad is the only arrangement its numbers permit.

That suggests an instrument, and the first half of it is built. The parent catalogues run to billions of tilings, but the subset lying inside the scale is small: **244,099 records**, some thirty megabytes, one row per dissection carrying its family, tier, order, dimensions, extremes, mean and median interval alongside the element sides themselves. That table is deposited with this paper. What remains is the module that queries it, and such a module would work in both directions.

Forward, from data to picture. Given a measured distribution, find the dissections that most nearly match it in population, range and shape. Exact matches will be vanishingly rare — section 5.3 explains why — so this returns the closest representable analogue rather than an identity. It is a way of asking what a set of percentages would look like if it could be drawn.

Backward, from picture to data. Given a specification — a population of 26, both extremes present, a median reaching fifty — return the distributions that satisfy it. Here the match is exact, because the element set of the dissection *is* the distribution, guaranteed drawable and guaranteed to carry the statistics asked for. This is the more powerful direction, and it is the one the maths teacher of section 1 used: he chose the dissection first and built the test to land on it. Run against the present census, that particular query returns exactly one row.

Building the query layer needs no new mathematics and no new computation — the table is done, it fits on a telephone, and every figure in it has been rebuilt from the published catalogues rather than copied. I offer it as the open proposal of this paper, and would be glad to see someone take it up.

4.3 Twenty-six, and four hundred and forty-one

The smallest hectoquad sits at order **26**. That is the magic constant of the hexagrams in my *Star Face* model; it is the number of characteristic elements of the cube, six faces, eight vertices and twelve edges; it is the number of times the cube's thirteen rotation axes cross its surface; it is the only positive integer wedged between a square and a cube, 25 and 27; it is the atomic number of iron; it is the count of letters in the English alphabet; and it is the gematria of the Tetragrammaton, yod 10, hey 5, vav 6, hey 5.

And the order-26 population is **441** squares. Now, $441 = 21^2$, and 21 is the lowest order at which a perfect squared square exists at all — Duijvestijn's, unique to its order. So the number of squarings at the order where hectoquads begin is the square of the order where squarings begin. 441 is also the gematria of *emet*, אמת, truth: aleph 1, mem 40, tav 400 — in the tradition the seal of God.

I claim no mechanism, and anyone offering one should be disbelieved. Duijvestijn counted 441 with four workstations running through the nights of early 1993, and the count answers to nothing but the mathematics. But the coincidence is what made me look at squared squares in the first place, and it is honest to say so.

There is a smaller echo of the same thing. The painting on the title page is of Duijvestijn's order-21 square, which is also the emblem of the Trinity Mathematical Society, founded at Trinity College, Cambridge in 1919 by G. H. Hardy — and the dissection was first sequenced in the Online Encyclopedia of Integer Sequences as A014530, which is where I met it. The painting is now cited in both standing references for the subject: among the links of A014530, approved personally by Neil Sloane on 13 March 2025; and at squaring.net/links/links.html, where Stuart Anderson shows it as what it was made to be in 2022 — a present for the fiftieth birthday of my friend, the Hungarian-American artist Tamás Vészi.

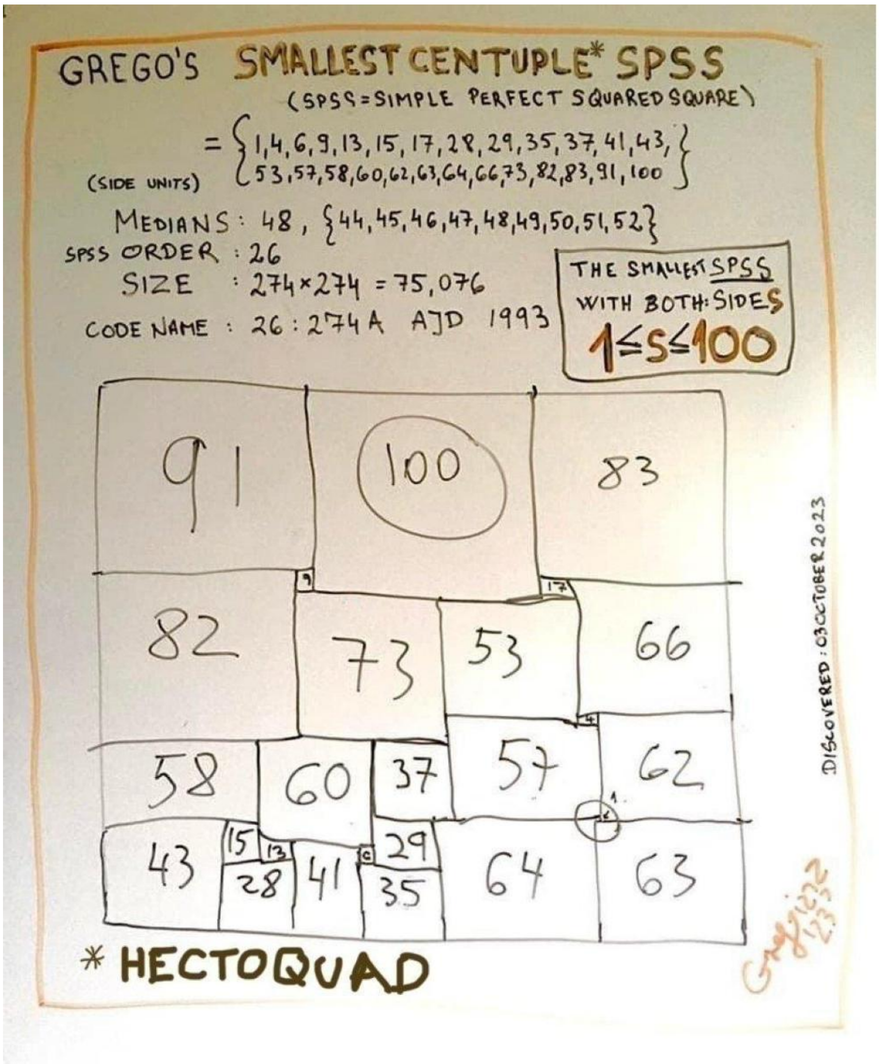
5 The answer to the problem

First half — could the students do it? Yes, and that is the surprise. A class in which everyone scored differently, one pupil scored 1 and one scored 100, and whose size equals the number of squares, is precisely a hectoquad. They exist. The smallest is 26:274A, so a class of 26 can do it. The teacher's first condition is met.

Second half — did a class pass? Take the class of 26 and apply the three sieves of my *Star Face* model to the statistic itself.

Sieve	Statistic	Result
unambiguity	mode	**none exists.** Everyone performed differently, so every score occurs exactly once and no value is more common than another. The mode is sieved out for every class in the problem
equilibrium	mean	**45.77.** Below fifty. This class fails on the mean
reliability	median	**a range, not a number.** With 26 pupils there is no middle pupil. Any value strictly between the 13th and 14th scores, 43 and 53, divides the class exactly 13 and 13 — so the median is the set {44, 45, 46, 47, 48, 49, 50, 51, 52}

Table 4. The three sieves applied to the class of 26.



The author's working sheet, 2023, where the class was still called centuple. The medians are written along the top: the question of what a median is for an even population is the one the whole problem turns on.

Two different things are at work and they should not be confused. The median is unmoved by the 1 and the 100 exactly as the reliability sieve promises — no outlying score can shift it. What leaves it undetermined between 43 and 53 is *parity*: an even population has no middle member, so the median is an interval rather than a point. Robust and undetermined are not opposites here.

The convention of quoting (43 + 53) / 2 = 48 as “the” median is a reporting habit, not a definition. The median is whatever splits the population in half, and for an even-numbered class that is an interval. **Fifty lies inside it.**

So the class of 26 did not fail: its median reaches fifty. Both of the Director's conditions are satisfied by that one class — and provided that class is the junior one, which section 5.2 shows to be the only size at which this can happen, **the maths teacher got his pay raise**.

The parity is doing the work. An odd class has a single middle pupil and therefore one fixed median, with no room to argue; an even class has an interval. It may be why the problem gives two classes rather than one, the most junior and the most senior. Whether that was meant to echo the hectoquads, which run from order 26 to order 37, I leave to the reader — though a class of 37 would be odd-numbered, and so would have a single fixed median with no interval to argue over.

5.1 The class of 26 in full

Statistic	Value	
pupils (order)	26	even, which is what makes the median an interval
side of the chalk square	274	$\Sigma s^2 = 75{,}076 = 274^2$
lowest / highest score	1 / 100	both extremes present, as the problem requires
total of the scores	1,190	
mean	45.7692	fails the Director's fifty
median	44 to 52	nine admissible values; **fifty is one of them**
mode	none	no score is repeated, so none exists
standard deviation	28.16	the spread the 1 and the 100 force
ceiling on the mean	53.74	$S / \sqrt{n}$, reached only if all sides were equal

Table 5. Everything the class of 26 can be said to have scored.

The last two lines are worth a moment. Because the areas must total the whole square, the mean m, the order n, the side S and the variance v are locked together:

$$m^2 = S^2 / n - v$$

which for this dissection reads 2094.8 = 2887.5 − 792.7. The mean is the ceiling less the spread. A hectoquad is obliged to hold both a 1 and a 100, so it is obliged to be spread, and the spread is subtracted from whatever the ceiling allows. Here the ceiling was 53.74 and the spread took 7.97 of it. That is why the mean falls short of fifty and the median does not.

5.2 Why the wording is exact

The census settles something the answer alone does not: that the problem is well posed, and that not one clause of it is decoration. Each condition in the wording cuts the field, and removing any of them would leave the answer ambiguous.

The wording	What it forces	Left standing
(the scale itself)	every element side between 1 and 100	244,099
one scored 1, one scored 100	both extremes present — a hectoquad	4,866
everyone performed differently	no element repeated — the dissection must be perfect	31
the two squares he had drawn with chalk	a square outline, not a rectangle	**8**
at least fifty in the statistically weighed overall score	the surviving statistic must reach fifty	**1**

Table 6. The conditions of the problem read as successive sieves over every dissection inside the scale. The two extremes cut 244,099 to 4,866; perfection removes 4,835 of those; the square outline removes the 23 rectangles; fifty removes seven of the remaining eight.

A quarter of a million dissections enter, and one leaves. The wording is not a riddle that happens to have an answer — it is a specification with a unique solution, and the census is what demonstrates that.

What survives is singular twice over, in two ways that had no obligation to coincide. It is the smallest: no perfect squared hectoquad exists below order 26, the orders from 21 to 25 having been enumerated in full. And it is the only one whose scores reach fifty: of the eight, only 26:274A has a median interval containing fifty, the others sitting at 29, 26, 27, 29, 27, 42 and 38. It carries the highest average of the eight as well, at 45.7692 — the first member found is also the best-performing.

Nor could the two singularities have fallen anywhere else. The median reaches fifty only by being an interval, and it is an interval only because the order is even; and 26 is the lowest even order in the entire census. The smallest hectoquad and the only one that passes are the same object because the property that lets it pass is the property that puts it first.

So the raise turns on the smaller of the two classes. Both could tile the yard — every hectoquad order permits it — but only a class of 26 passes, and since 26 is the lowest order in the census, the class that earns the raise cannot be the senior one. The teacher gets his raise if and only if his smaller class has exactly 26 pupils and the Director reads the median as the interval it is.

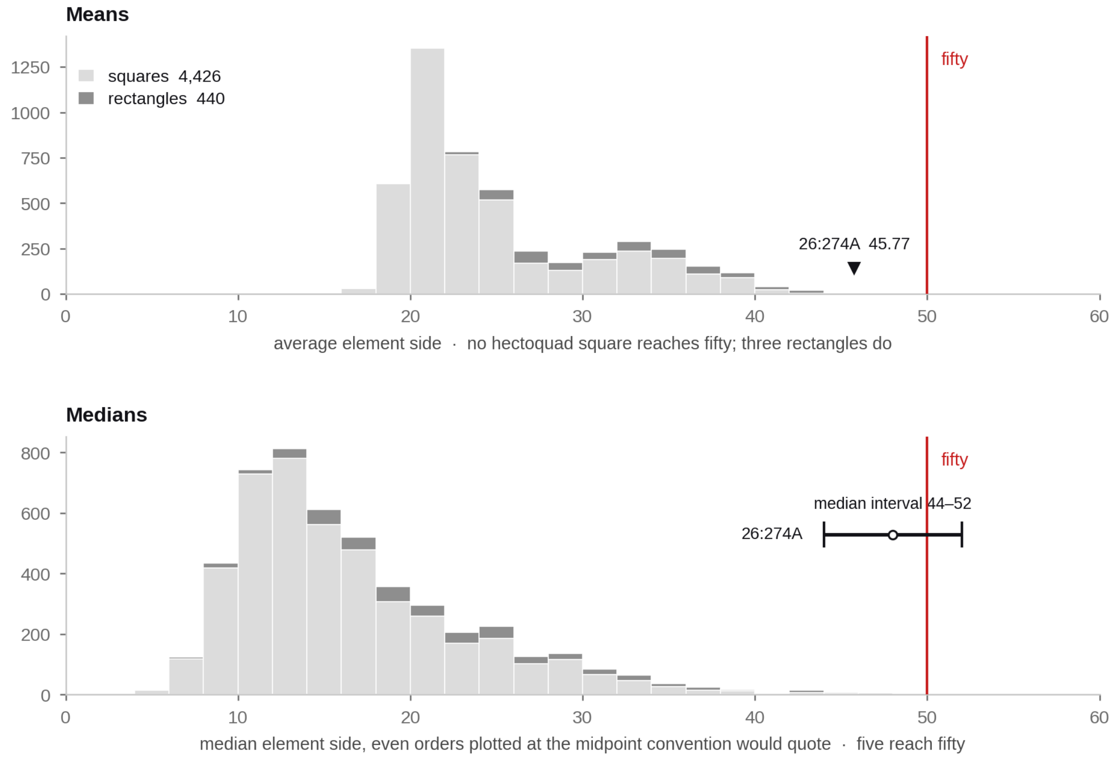


Figure 2. Every hectoquad in the census, by its average and by its median. The mass of both distributions sits far below fifty. Three rectangles pass fifty on the average and no square does; five reach it on the median, of which 26:274A is the only perfect squared square. Its median is drawn as the interval it is, 44 to 52, with the open circle marking 48 — the midpoint that convention would quote, and which falls short.

5.3 The improbability, and why it does not matter

An objection should be met here rather than left for the reader to find. “Everyone happened to perform differently” sounds like mild luck, but the story needs far more than distinctness: it needs twenty-six scores that are exactly the element set of a hectoquad. There are some 7×10^{23} ways to choose 26 distinct integers from 1 to 100. Six of them tile a square, and just one of those six holds both a 1 and a 100. Read as chance, the scene is not unlikely but absurd.

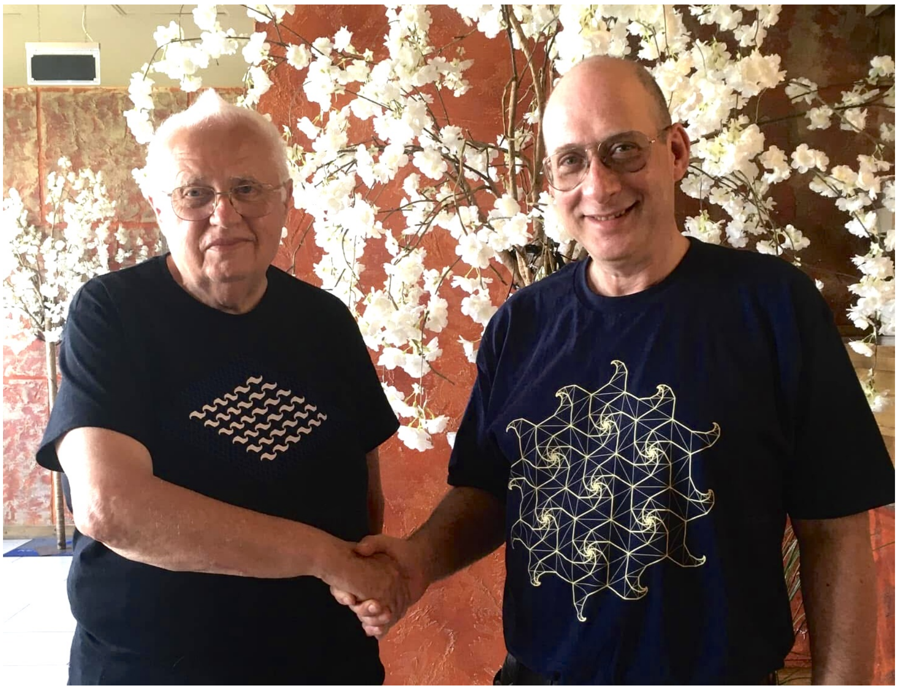
It is not chance. The protagonist is a mathematics teacher who knows his pupils, knows the size of his classes, and knows the census — including the order and the median of its smallest member. He set a paper of worded problems to be composed and solved in full, an instrument with room to score anywhere on the scale, difficult enough to wall the absentee and open enough to let the ablest reach a hundred. He was not falsifying marks; he was designing a measurement whose outcome he could place. Every pupil earned the score they were given.

And the census shows he had no room to manoeuvre. Of the eight, only 26:274A has a median reaching fifty, so only a class of twenty-six could ever have earned him the raise. He did not pick a convenient target from several; the census left him exactly one, and it happens to be the smallest that exists.

The answer is robust in any case. Because everyone scored differently, both dissections must be perfect, so whatever the senior class turns out to be, its order lies in 29, 30, 31, 33, 35 or 37 — and not one of those has a median reaching fifty. The contrivance sets the scene; it does no work in producing the conclusion. That could not be said before the counting was done.

5.4 The one solver

The problem was put to a good many people, in person and in writing, and published for anyone who cared to attempt it. Of all of them, exactly one solved it straight away: **Professor Lajos Szilassi**, distinguished geometer, whose name is on the Szilassi polyhedron — the only polyhedron besides the tetrahedron in which every face shares an edge with every other. We met in Szeged on 23 June 2023.



Professor Lajos Szilassi and the author, Szeged, 23 June 2023. Szilassi was the only person to solve the problem of section 1 immediately.

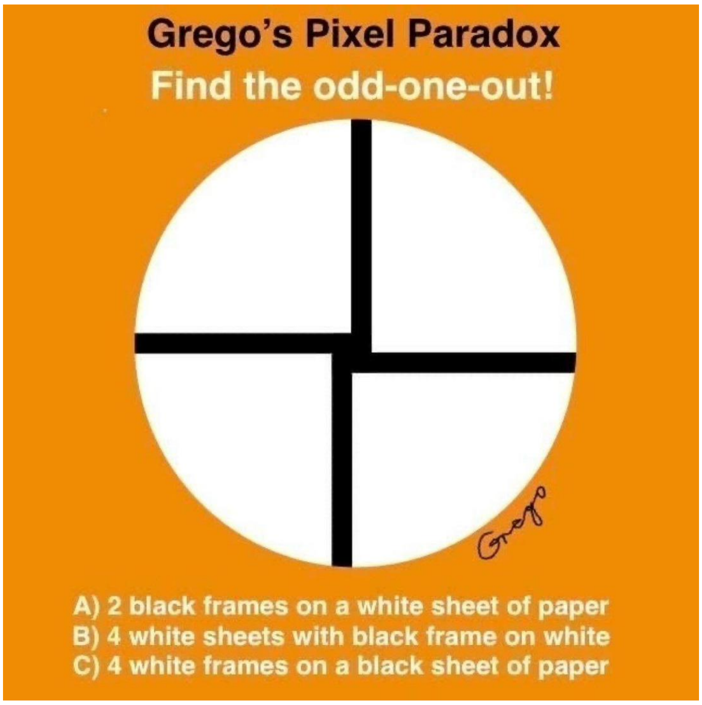
A stainless steel model of his polyhedron has stood at Pierre de Fermat's house in Beaumont-de-Lomagne since 2002, and the form now stands as public art on three continents. On 26 March 2026 the world's first glass Szilassi polyhedron was unveiled in front of the József Attila Study and Information Centre of the University of Szeged, where Szilassi was first a student and then a teacher for more than four decades. It is the work of the glass artist Dorka Borbás with the architects Péter Vesmás and Tamás Szögi. He said at the unveiling that he had never dared dream a statue would be raised to the polyhedron that bears his name.

That one solver is worth recording for what it says about the problem rather than about him. A puzzle that yields to one reader immediately and to no one else is not merely hard; it is asking for something specific. What it asks for is the habit of treating a median as a thing that halves a population rather than a number to be quoted — and of treating a dissection as an object to be read rather than seen. A geometer who is also a schoolteacher has both habits professionally, and Professor Szilassi spent his career with both.

The protagonist of the problem is a mathematics teacher. So was the only person who saw through it at once.

6 The Pixel Paradox

A hectoquad containing a unit square is difficult to draw honestly. At any reasonable size the 1 is a single pixel, and a line drawn to bound it is thicker than the thing it bounds. The border of a square and the square itself become the same mark. I set the difficulty as a puzzle.



Grego's Pixel Paradox. A drawing problem that is also a question about what a boundary is.

6.1 Why it happens

A dissection is a set of squares, but a drawing of it is a set of *lines*, and lines have width. Draw 26:274A at a sensible size on a page — say 300 points across — and one unit of side becomes 1.09 points. A hairline rule is about half of that. So the unit square is drawn as a mark roughly the size of its own border, and the eye can no longer tell the square from the frame around it. Enlarge the figure and the problem does not go away: the border grows with the square only if you scale the line weight too, and if you do not, the 100 becomes a field and the 1 becomes a smudge.

This is not a defect of the drawing but a property of the object: a hundredfold range is built into the definition, and no single reproduction serves both ends of it. The catalogues omit the label on the smallest elements; Figure 1 leads them out to the margin, which is an admission rather than a solution.

The paradox above puts the same difficulty as a question with three answers offered and no way to choose between them from the image alone. Whether you are looking at black frames on white, or white sheets edged in black, or white frames on black, depends entirely on which marks you call the object and which you call the ground — and a unit square inside a hectoquad sits exactly on that line.

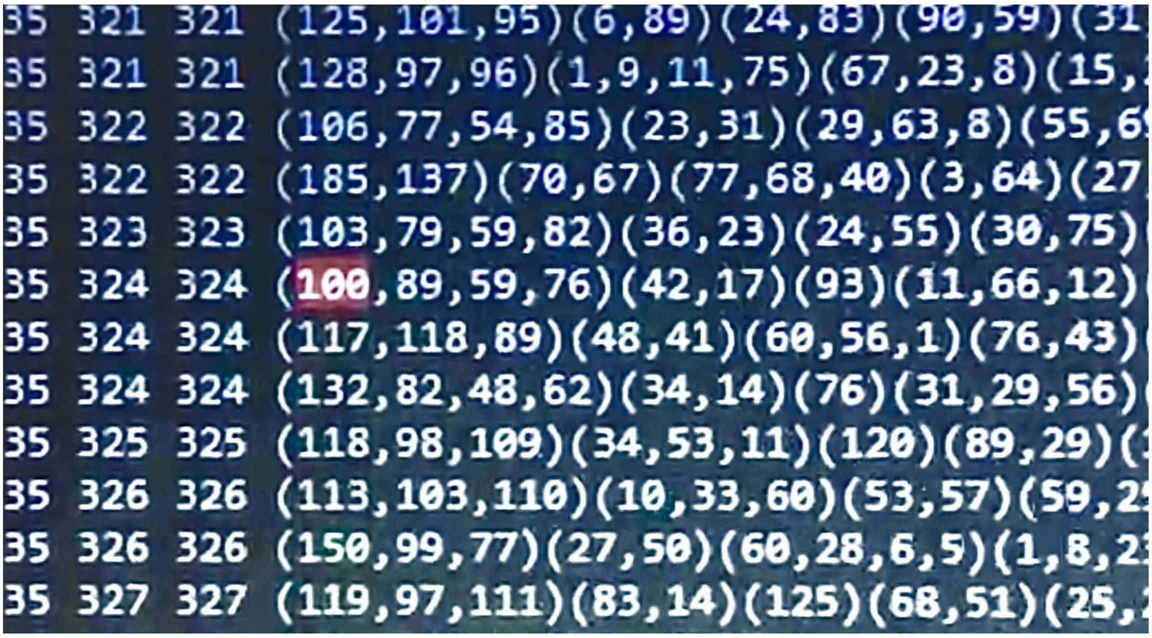
The odd one out is B — and not because it reads differently. B is the only one the picture cannot actually be. Lay four white sheets, each with a black frame, on a white ground, and at the exact centre where their four corners meet the ground must show through: one white point, at the crossing. Look as closely as you like. Zoom as deep as you like. That point is not there, and it never will be, because the rules that would bound it are wider than the gap they would leave.

The proof of it is in the literature. Stuart Anderson's catalogue of order-26 squared squares gives 26:274A with every element drawn and labelled — and **the 1 is not visible at all**. Where it should sit, four squares meet at what looks like a clean crossing of two rules. The element is in the data, in the Bouwkamp code, in the area sum; it is simply not in the picture, because at that scale its side is narrower than the line drawn to bound it. The standard reference for these objects cannot show the thing it is cataloguing.

That absent white point is the hectoquad's 1. Precisely the same difficulty: an element one hundredth the size of the largest cannot be shown when the line that bounds it is thicker than the thing itself. The paradox is not a trick of perception but a limit of representation — and it is why the smallest square in Figure 1 had to be led out to the margin on a leader rather than labelled where it sits.

And that is how the students could only find it. They were permitted a calculator, a library, the internet — and what they reached was the catalogue, whose picture does not show a 1. They could not see the answer. They could only read it: from the Bouwkamp code, from the list of elements, from the data rather than from the drawing. A class that trusted the picture would have reported the task impossible, and been wrong.

Which is the whole of it. What is shown and what is there have parted company, and only the numbers still hold. That is true of the unit square in the catalogue, and it is true of a class average that reports 45.77 for a population whose median reaches fifty.



Screenshot from a public video documenting how the author found the largest known hectoquad, **35:324A JBW 2016** — watch at tinyurl.com/hectoquad35. Each line is one dissection: order, side, then its Bouwkamp code. The search reads every row for a largest element of exactly 100 with a 1 present somewhere in the same row. Nothing is seen; everything is read.

The video is a complete record rather than an illustration: the author on camera and in his own voice, the software in use, the search method stated, and the finding itself as it happens. Mathematics rarely films its own discoveries, and for a class defined by a search rather than a proof, a record of how the search was actually conducted seems worth having — both as provenance for the claim and as an answer to anyone who would like to repeat it.

6.2 A sailor's million-dollar remark

The worded puzzle and the paradox hidden inside it do more than outline the intended sieve model. They underline the fallacy so often met in everyday statistical reporting — a fallacy that translates into cash in commerce, and into lives in medicine.

It is like running cruises from Los Angeles to London and candidly crossing the International Date Line east to west every time at exactly midnight, when the guests and ninety per cent of the crew are asleep. A day is lost at sea. Nobody aboard notices it go. And a lost day, multiplied by occupancy, is hidden income with a four-digit multiplier.

The class of 26 is the same manoeuvre in miniature. Report the mean and the class failed at 45.77; report the median and it passed. Neither figure is false. The choice of statistic was made before the arithmetic began, and it decided the answer — which is exactly what nobody watching the number is looking at.

7 A hypothetical sequence entry

Set out here in the style of the Online Encyclopedia of Integer Sequences®, founded by Neil Sloane in 1964. It is hypothetical: the entry is drafted, not submitted. The dissection painted on the title page is sequenced there as A014530, where I first met it.

Field	Entry
NAME	Side lengths a(n) in the lowest order (26) example of a perfect squared square with 1 ≤ a(n) ≤ 100 and both extreme values present
DATA	1, 4, 6, 9, 13, 15, 17, 28, 29, 35, 37, 41, 43, 53, 57, 58, 60, 62, 63, 64, 66, 73, 82, 83, 91, 100
OFFSET	1, 2
COMMENTS	a(n) are the element sides of 26:274A (A. J. W. Duijvestijn, 1993), the lowest-order squared square whose sides all lie in [1, 100] with both extremes present. Such a dissection is here called a *hectoquad*. Exactly 8 simple perfect squared squares are hectoquads in orders 21 to 37, the complete range of enumerated SPSS: 26:274A; 29:221A and 29:264A (A. J. Palmer, 2011); 30:246A (J. D. Skinner); 31:228B (2013); 33:279A (2013); 35:324A (2016); 37:317D (2020) — the last four J. B. Williams. Their means are 45.7692, 33.5517, 38.2069, 34.8667, 33.1613, 37.1818, 45.1429 and 43.9730; none reaches 50. Order 26 is minimal: orders 21 to 25 are enumerated in full (1, 8, 12, 26 and 160 SPSS) and contain none. Order 26 is even, so the median of a(n) is not a single value but the interval 44 to 52, which does contain 50. The perfect part of the class is finite. The elements are distinct integers not exceeding 100, so the order is at most 100; and since the area cannot exceed 1^2 + 2^2 + ... + 100^2 = 338350, the side is at most 581. Orders 38 to 100 are not enumerated, so 8 is exact over orders 21 to 37 and a lower bound for the class. Compound perfect squared squares of orders 24 to 39 include no hectoquad. Admitting repeated element values or rectangular outlines gives 4858 further members in the published catalogues — 4418 simple imperfect squared squares, 417 simple imperfect squared rectangles and 23 simple perfect squared rectangles — the smallest of all being the order-11 rectangle 11:185x168A.
REFERENCES	N. J. A. Sloane and Simon Plouffe, *The Encyclopedia of Integer Sequences*, Academic Press, San Diego, 1995, Fig. M4482. — I. Stewart, *Squaring the Square*, Scientific American 277, July 1997, 94–96
LINKS	Stuart E. Anderson, squaring.net/sq/ss/spss/o26/spsso26.pdf, *Catalogue of Perfect Squared Squares, Order 26*, p. 16 Stuart E. Anderson, squaring.net/sq/ss/spss/spss.html, *Catalogues of Perfect Squared Squares*, orders 21 to 37, from which the census above is drawn Gergely Földvári, grego.hu/star, *Star Face — Grego's Polyhedral Sieve Model for Medical Diagnosis*, p. 73 The Joy of Thinking Foundation / Gondolkodás Öröme Alapítvány, mindenkijon.hu/p/mindenkijon-19-nap, nationwide open online mathematical problem-solving project *minden·ki·jön*, Day 19, paragraph 2, 7 May 2025 Trinity College Mathematical Society, tms.soc.srcf.net, *The Squared Square* Wikipedia, *Squaring the square*
EXAMPLE	Bouwkamp code: 26 274 274 (91,100,83) (17,66) (82,9) (73,53) (4,62) (57) (58,60,37) (29,64,1) (63) (43,15) (13,41,6) (35) (28)
CROSSREFS	A014530
KEYWORD	nonn, fini, full
AUTHOR	Gergely Földvári

Table 7. The hypothetical entry in full.

7.1 On general interest

A sequence earns its place partly by being of interest beyond its author, and I would argue this one has shown it. The problem in section 1 — to which a hectoquad is the only possible answer — was set nationwide on 7 May 2025 by the Joy of Thinking Foundation, backed by the Alfréd Rényi Institute of Mathematics. A national mathematics foundation putting the question to the public is evidence that the object behind it is worth cataloguing — and it is evidence that existed before any paper was written about it.

8 What is settled, and what is not

Earlier drafts left three questions open. Two are now answered, and the third in one direction only.

Settled: order 26 is minimal. Orders 21 to 25 have been enumerated exhaustively — 1, 8, 12, 26 and 160 simple perfect squared squares — and none of the 207 is a hectoquad. Allowing imperfect and rectangular members lowers the floor rather than raising it: the smallest hectoquad of any kind is the order-11 rectangle 11:185×168A.

Settled: there are eight, not seven. An earlier draft listed seven and conjectured there were no more; that was wrong on both counts. Sweeping orders 21 to 37 gives eight, the eighth being 37:317D, found by James Williams in 2020. The same sweep corrected 31:228A to 31:228B, whose largest element is 88.

Settled: no hectoquad square averages fifty. This was the question I most wanted answered. Of the 4,426 hectoquad squares the highest average is 49.375, and among the eight perfect ones it is 26:274A's own 45.7692. Three rectangles do pass fifty — but the teacher drew squares on the yard. Over the whole census the median is not merely the correct route to the answer of section 5; it is the only one.

Not settled: whether that holds beyond the catalogues. The perfect part of the class is finite — order at most 100, side at most 581 — so the question is decidable, and one day it will be decided. But simple perfect squared squares are enumerated only to order 37, imperfect ones to 32, rectangles to 21 and 22; everything above is uncounted. So all three results are exact over the enumerated range and conjectural beyond it, and I would rather say so than let a census be mistaken for a proof.

What would close it is an argument rather than more computation. For 26:274A the ceiling on the mean is 53.74 and the forced spread caps it at 51.95, so fifty was available and not taken. The census says it does not happen; a proof would say it cannot, and I would be glad of company in looking for one.

References

Duijvestijn, A. J. W.	*Simple perfect squared square of lowest order*, J. Combin. Theory Ser. B 25 (1978), 240–243
Duijvestijn, A. J. W.	*Simple perfect squared squares and 2×1 squared rectangles of order 26*, Math. Comp. 65 (1996), 1359–1364
Anderson, S. E.	Catalogues of Perfect Squared Squares, orders 21 to 37 — squaring.net; the author's painting at squaring.net/links/links.html
OEIS®	A006983, simple perfect squared squares by order; A014530, whose links include a photograph of the author's painting of the order-21 square; A051596, standard Hebrew gematria
Stewart, I.	*Squaring the Square*, Scientific American 277, July 1997, 94–96
Sloane and Plouffe	*The Encyclopedia of Integer Sequences*, Academic Press, 1995, Fig. M4482
Trinity Mathematical Society	*The Squared Square* — the society's emblem; Trinity College, Cambridge, 1919
Joy of Thinking Foundation	*minden·ki·jön* Day 19, 7 May 2025, mindenkijon.hu
Földvári, G.	*Star Face*, grego.hu/sf, p. 73 — where the hectoquads first appear

Acknowledgements. Special thanks to **Stuart Errol Anderson** of New Zealand, who congratulated me on the class in our personal correspondence, when it still went by the name *centuples*, and who advised me to seek the full census. Section 4 is the answer to that advice, and it rests on his catalogues at squaring.net — as it does on the enumerations of A. J. W. Duijvestijn, J. D. Skinner, James Williams, Lorenz Milla, Stephen Johnson and Ed Pegg Jr.

Version 1.0. The class was established during the *Star Face* work of 2023–2026; the census of section 4 was run against the squaring.net catalogues as they stood on 3 September 2026, and every dissection quoted here was rebuilt from its Bouwkamp code before publication. Should the enumerated range widen, a later version will say so.

Grego — Gergely Földvári, Budapest, 3 September 2026.