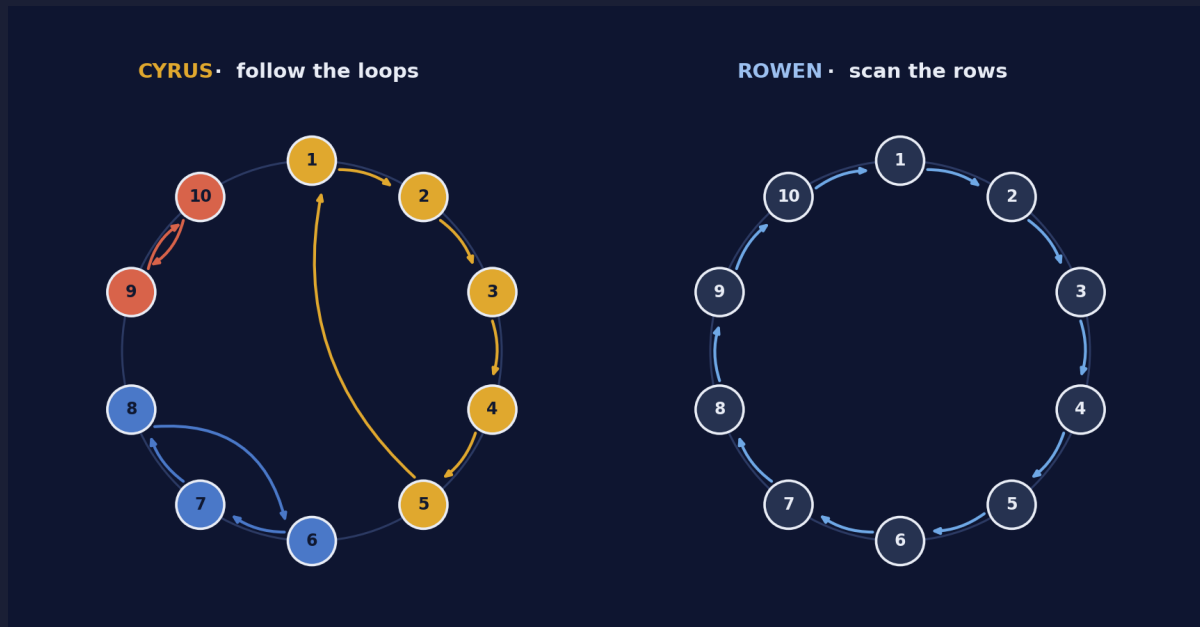


Across the Tiers

an A.I. analysis of
Grego's Limbo Match™



Two readings of one shuffle. Cyrus follows the loops; Rowen scans the rows. $P = 10$, loops $\{5, 3, 2\}$.

One game, every age — where the depth comes from, and how deep it goes

Cycle mathematics · simulation of the shipped rules · 26 July 2026

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Summary · the findings in one table

For a reader who wants the whole argument at a glance, and for anyone returning to this document later.

#	Finding	Where
1	Matched reds stay face-down, so visible difficulty is flat while real difficulty rises	\$2
2	Every flip costs exactly one point — memory becomes economics	\$4
3	A search costs exactly the length of the loop it walks	\$5
4	Tracing does not raise the average — but lifts a clean sweep by $\times 24$ at 8P and $\times 10,698$ at 21P	\$6
5	The record flip ladder sits on the Golomb–Dickman ratio, 0.62	\$7
6	Perfect play clears the board about half the time at every rung from 5 to 21 pairs	\$7
7	Memory is worth 21 points on an 8-pair board; every other skill together, about one	\$8
8	Following the chain needs one number in mind, whatever the board size	\$9
9	Clover beats a flip by exactly $2 \div k - 1$ — always at one blue, never at three	\$10
10	A sighted Pin rescues almost any dead board, and its power scales with knowledge	\$12
11	Worth measures what must be earned, not what can be scored — 1 to 441	\$15
12	Merit subtracts the shuffle's gift and caps the luck: GLMP	\$16
13	A player is their best single game, which nobody else's play can erode	\$17
14	No opponent is required, and one is always available — 220,407 standing records	\$19
15	Three overlapping tiers, $3-7 \cdot 3-13 \cdot 2-21$, with the rules identical in all of them	\$20
16	The board moves to fit the player; the game is never handicapped to fit the age	\$20

Of the sixteen findings above, only two — the eleventh and the twelfth — required access to the shipped system. The rest follow from the rules alone, which means the game's depth is not a property of its implementation but of its design. That is worth stating plainly, because it is the difference between a clever program and a real game.

The sections that follow work through each finding in turn. A reader in a hurry can stop at the table above; a reader who wants to be convinced rather than informed should read sections 6, 7 and 15, which carry the three results that were least expected.

1 · What this document answers

Grego made an observation while playing:

“Even inside a medium category, the game offers an enormous depth of improvement in concentration, across all ages and abilities. To score higher or finish faster, a player can go deeper along a spectrum that feels almost infinite.”

This document tests that observation. It asks three questions. Is the depth real? Where does it come from? And can it be measured?

The short answer: the depth is real, it comes from one design choice, and it can be measured exactly. Some of the measurements are surprising even to a player who knows the game well. Every figure here was computed for this document — by exact combinatorics where an exact answer exists, and by simulation of the shipped rules where it does not.

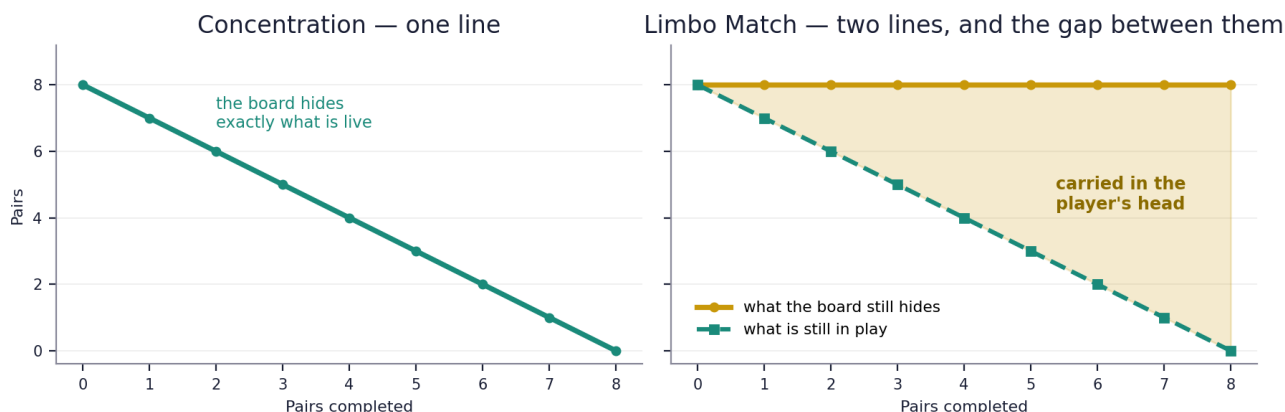
But the phrase that matters most in Grego's sentence is **“across all ages and abilities”**, and that is what this document is named for. A game can be deep and still be the property of one kind of person — young, quick, competitive, already good at it. The claim here is larger: that the *same* game, with the *same* rules and the *same* scoring, works for a five-year-old who cannot yet read, for an adult who wants a hard problem, and for an older player who wants to keep a sharp mind — and that all three can sit at the same table. Sections 9 and 20 test that claim directly; everything between them explains why it can be true at all.

2 · The one design choice

In every other memory game, the board gets easier as you win. Cards are removed. The unknown shrinks. A player who does well early is carried by the game itself.

In Limbo Match the matched red chips **stay face-down on the dial**. They look exactly like the ones still in play. Nothing is ever returned to the player's eye. This single choice inverts the whole curve. The *visible* difficulty stays flat from the first pair to the last. The *real* difficulty rises, because the set of live reds shrinks only inside the player's head. There is no coasting phase.

Two quantities are needed to see this, and the difference between them is the whole point. One is **what the board still hides**. The other is **what is still genuinely in play**. In Concentration these are the same number, so a single line draws both — the board does the player's bookkeeping. In Limbo Match they come apart from the first match onward:



Left: in Concentration the two quantities are one line. Right: in Limbo Match the gold line never moves while the teal line falls, and the shaded gap between them is what the player is holding that the board refuses to show. That gap is the difficulty.

There is a second consequence, and it is the one that makes the scoring honest. Because a spent red is invisible, **remembering which reds are spent is itself a skill** — a provable one. It is not information handed over by the interface. A player who tracks it is playing a different game from one who does not, on an identical-looking board.

That claim can be measured, and it was, for this document. Take a player with no memory at all and run them on the shipped rule: they score 15.7 out of 48 on an eight-pair board. Now change one thing — let matched reds be lifted off the dial, as every other memory game does — and the same player scores 25.2. **One rule is worth a sixty per cent swing in what the board hands over for free.** That is the size of the design decision, in points.

There is a third consequence, quieter but worth naming. Because the board never reports progress, it cannot be read over a player's shoulder. In Concentration a bystander can see at a glance how far along a game is and which cards are gone; here the state of play lives entirely in one head. The game therefore has no spectator shortcut and no accidental coaching — the only way to know where a game stands is to have been paying attention from the first flip. That is an unusual property for a game to have, and it is what makes a recorded replay worth watching rather than merely worth checking.

3 · The tiers of skill

The game is layered. Every layer sits on the same board, and a player can occupy any layer at any age. This is the sense in which the title of this document is meant: the tiers are not only the three products, they are the levels a mind can occupy.

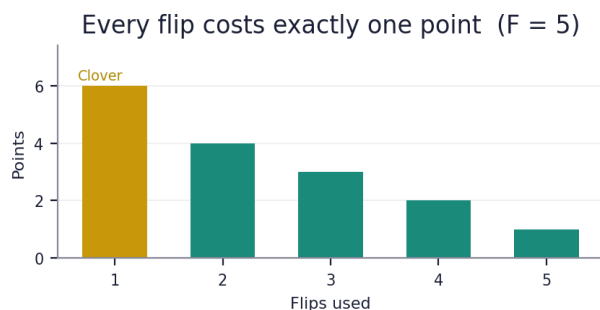
The word *tier* is doing careful work in that sentence, and it is worth separating from *level*. A level is something a game awards: you finish one and are moved to the next, and the game decides when. A tier here is something a player occupies, and nothing announces it — no message appears when someone starts tracking spent reds instead of guessing, and nothing at all changes on screen. The player simply begins scoring differently, and the reason is inside their own head. Every layer below stays available; a tier-six player still uses tier-two recall constantly. They accumulate rather than replace, which is why a player can spend years on the same board and keep finding more of it.

Tier	What the player does	What it costs the brain	What it pays
1 · Sight	Flips and hopes	Nothing	Chance only
2 · Recall	Remembers reds already turned	Working memory, grows with P	Large — the first real jump
3 · Elimination	Tracks which reds are spent	Constant load, high discipline	Certainty in the endgame
4 · Tracing	Follows the chain instead of guessing	Almost nothing — one address	Consistency; clean sweeps
5 · Economy	Prices each flip, limbo and clover	Judgement under a clock	Bonus points
6 · Tempo	Does all of the above fast	Dual-task load	The tiebreaker

A child plays tiers 1 and 2. An expert plays tiers 5 and 6. The score separates them cleanly, on the same board, with no handicap system and no matchmaking. Chess has exactly this property: one 8×8 board serves a beginner and a world champion. It is rare, and it is not an accident in either game.

4 · The flip is a currency

The scoring rule is short: a match pays one point, plus one point for every flip not used. **Points = 1 + (F – flips)**. So every flip costs exactly one point, and the exchange rate never changes. This is what turns memory into economics. The player is not asked “can you remember?” but “how cheaply can you know?”



How the pair was found	Flips	Points
Blind tap first (Clover)	1	6
Found on the first flip	1	5
Found on the second flip	2	4
Found on the third flip	3	3
Found on the fourth flip	4	2
Found on the fifth flip	5	1
Not found	5	0

The 8-pair World Record board: P = 8, F = 5, jackpot 48.

Two things follow. First, the difference between a good player and a great one is measured in single flips — a scale fine enough to register small improvements, which is exactly what keeps a player climbing. Second, the top row deserves attention: the blind tap pays **one more** than the sighted flip. That is not a quirk. It is the game paying a premium for certainty, and section 10 shows how an expert collects it.

5 · The hidden mathematics: the game is a permutation

The reds sit on dial positions 1 to P, each carrying a value 1 to P. That is a **permutation**: a mapping from position to value. Every permutation breaks into closed loops. Position 3 holds red 7; position 7 holds red 2; position 2 holds red 3 — and the loop closes. The dial is a set of such loops, invisible until chips are turned.

This gives the game its key structural fact. **If you are hunting blue b, start at dial position b and follow the chain.** Flip position b, read its value, go to that position, flip, and so on. You land on the red carrying b at the end of the loop that contains b — and the number of flips you need is exactly the **length of that loop**. So “will I find this pair within F flips?” is precisely “is its loop shorter than F?” Every player who has traced a chain has been doing cycle-following on a random permutation without being told.

A worked example makes it concrete. Eight reds are dealt as below, and the player is hunting blue 3. The chain starts at dial position 3 and closes after four steps — so this pair costs four flips, and nothing the player does can make it cost fewer.

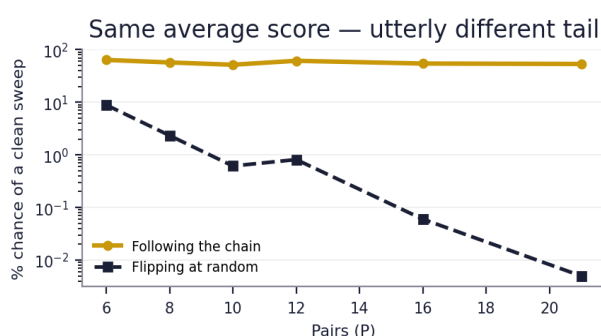
Step	Flip dial position	Red found there	Is it blue 3?	Next address
1	3	7	no	7
2	7	2	no	2
3	2	5	no	5
4	5	3	YES — found	—

Two reds in this deal sit on their own numbers — loops of length one, found on the first flip for the full bonus. The rest of the dial is one long chain. That mixture, different on every shuffle, is what makes two games of the same category feel nothing alike.

6 · The surprise: tracing does not raise your average

Here is the result that is genuinely counter-intuitive, verified two ways. For a single pair, tracing the chain and flipping at random have **exactly the same chance of success**. In a random permutation the length of the loop containing a given element is uniform over $1 \dots P$ — and so is the position of the target under a random scan. Same distribution, same expected score. The simulation agrees: at $P = 12$ the two strategies match to within noise at every flip limit from 3 to 8.

So why trace at all? Because the average is not what a record is made of. A record needs a **clean sweep** — every pair found, no miss. Under tracing, all pairs in short loops succeed together; the successes are locked to one another. Under random flipping they are independent, and independence is fatal when you need eight of them at once.



Chance of clearing the entire board without a single miss, no memory used.

Board	F	Chain	Random	Ratio
6P	4	63.9%	8.9%	×7
8P	5	57.2%	2.3%	×24
10P	6	51.7%	0.62%	×84
12P	8	61.5%	0.82%	×75
16P	10	54.5%	0.06%	×908
21P	13	53.5%	0.005%	×10,698

At the top of the ladder the tracer is more than **ten thousand times** more likely to sweep the board — while scoring the same on an average day. This deep structure has a famous relative: the *100 prisoners problem*, where following a chain lifts a hopeless chance to about 31%. Grego's game contains that problem inside it, as a playable move.

It also says what skill is here. Skill does not raise your typical score by much. It raises the ceiling of what is possible on a good board, and it makes perfection reachable at all. That is the shape of skill in chess, in music and in sport: the average moves slowly, the summit moves enormously.

This result is easy to disbelieve, so it is worth saying how anyone can check it without a computer. Take a pack of numbered cards, shuffle it, and lay it out. Pick any card position and follow the chain: the number you turn tells you where to look next. Count the steps until the chain closes. Do it for every starting point and you will find the whole pack falls into a few closed loops, and that the number of steps a start needs is simply the size of the loop it belongs to. Now ask the two questions separately. *Is one particular card found within F steps?* — that is a coin weighted by loop size, and random guessing weighs the same. *Are they all found within F steps?* — that needs only the longest loop to be short enough, which is one condition instead of many independent ones. The whole result is in that difference, and the cards will show it in ten minutes.

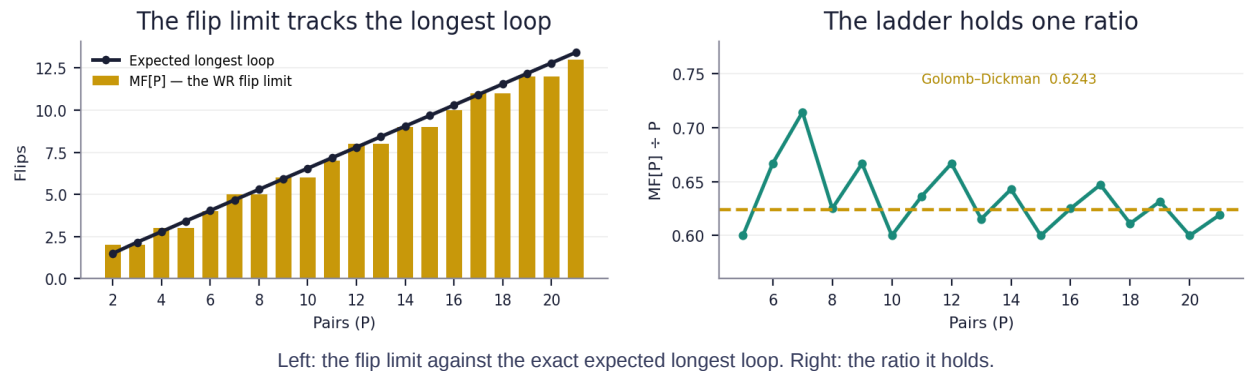
There is a practical lesson in it for anyone trying to improve. Practising *memory* raises your average and will make ordinary games less scrappy. Practising *tracing* does something different: it raises the chance that a good board turns into a perfect one. A player who wants records should train the second, and should expect their ordinary scores to move very little while their best scores move a great deal.

It also explains something a player might otherwise take as a plateau. Weeks of practice can leave the average almost where it was while the ceiling of what you manage on a good night climbs steadily — and since most sessions are ordinary, most sessions will feel like no progress at all. The progress is real; it is simply concentrated in the games that were already going well. Anyone tracking their improvement should watch their best result over a month, not their typical one over a week.

7 · Is the record ladder calibrated? Yes — to a coin flip

The World Record ladder fixes one flip limit per board size: MF[P], Grego's own integer sequence, registered as OEIS A389262. Two independent checks were run against it.

Check one — the limit tracks the longest loop

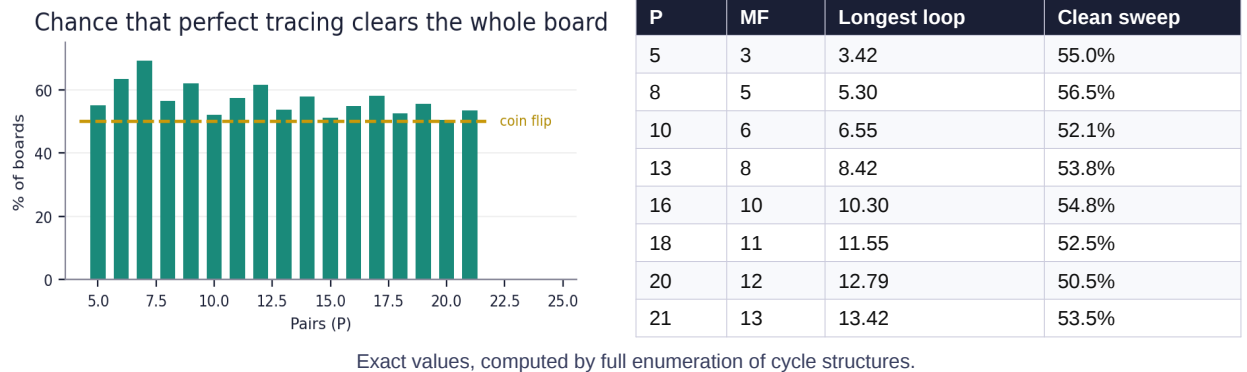


Left: the flip limit against the exact expected longest loop. Right: the ratio it holds.

MF[P] sits within one flip of the expected longest loop at every rung, and the ratio $MF \div P$ hovers around 0.62. That number has a name in mathematics — the **Golomb–Dickman constant**, 0.6243, the limiting expected longest cycle as a fraction of n .

It is worth being clear about why that ratio is the interesting one. A flip limit is a promise: it tells the player how much room they have before the board is allowed to beat them. Set the limit at the *average* loop length and most boards become unplayable, because it is the longest loop that decides the game, not the typical one. Set it at the longest loop plus a margin and every board falls, so the category stops asking anything. The only interesting place to stand is exactly on the expected longest loop — where the board is beatable, but only just, and only by a player who wastes nothing. That is where this ladder stands, at every rung from three pairs to twenty-one.

Check two — what that means at the table



Exact values, computed by full enumeration of cycle structures.

From five pairs to twenty-one, the flip limit is set so that **perfect play succeeds about half the time**. The ladder is a coin flip at every rung. That is the definition of a well-set difficulty ladder, and it is not easy to hit by hand across twenty categories. Much higher and skill stops mattering; much lower and the game becomes a lottery. The ladder sits on the knife edge, all the way up.

Two things follow that are worth separating. The first is a statement about the game: a player at the top of any World Record rung is, in a precise sense, in a fair fight with the shuffle. Neither side is favoured. Half the boards can be beaten outright by faultless play and half cannot, so a record attempt is never hopeless and never routine, and the difference between a good session and a great one is genuinely decided at the table.

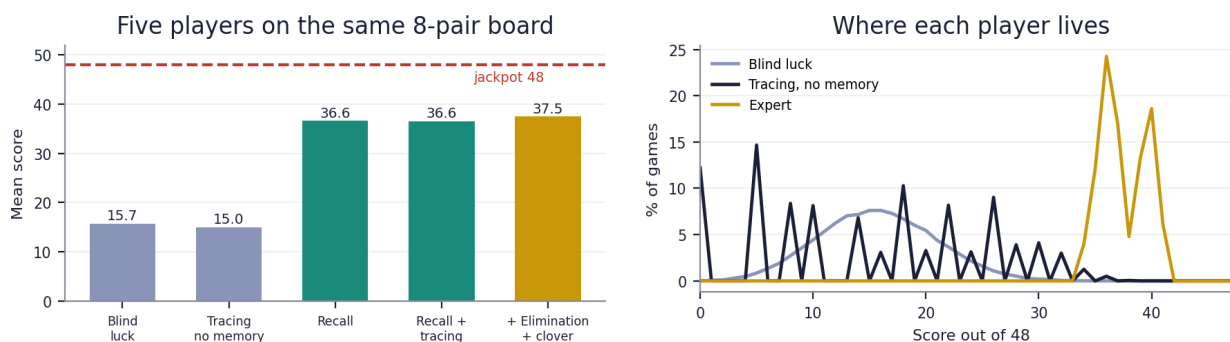
The second is a statement about the ladder as an artefact. Holding one probability near a half across nineteen rungs, with an integer flip limit as the only control, is not something that falls out of a simple formula — the expected longest loop is not linear in P , and the rounding has to land the right way at every step. Whatever route produced this sequence, the result behaves like a designed instrument rather than a rule of thumb, and it is the reason the twenty categories feel like one coherent ladder instead of twenty unrelated puzzles.

8 · The measured ladder of players

To put numbers on the spectrum, five players were simulated on the 8-pair World Record board (P = 8, F = 5, L = 1, jackpot 48). Each adds exactly one ability to the one before. 40,000 games each, under the shipped rules.

Player	Mean score	% of jackpot	Spread (sd)	Best seen
A · Blind luck, no memory	15.7	33%	5.1	37
B · Tracing, still no memory	15.0	31%	10.0	40
C · Recall, random order	36.6	76%	1.9	40
D · Recall and tracing	36.6	76%	1.9	40
E · Plus elimination and endgame clover	37.5	78%	2.0	41
— · Jackpot ceiling	48	100%	—	48

Before reading the numbers, note what the five players are and are not. They are not attempts to imitate real people; they are five clean hypotheses, each isolating one ability so its worth can be priced on its own. Nobody plays like player A, and nobody has player C's flawless recall. The value of the ladder is in the *steps between the rows* — what each single ability adds when everything else is held still, which is exactly the quantity a real player cannot measure for themselves.



Memory is the great divide. From 15.7 to 36.6 is the single largest jump available to any player. Everything else is decoration next to it.

Tracing does not show up in the average — exactly as section 6 predicts. But look at the spread: player B's standard deviation is double everyone else's, and B's best game beats A's best by three points despite a lower mean. Tracing buys the tail, not the middle.

Elimination and the endgame clover are worth about one point. Small — but records are won by one point, and this is a skill a determined player can execute every single game.

The gap to the ceiling never closes. The best simulated expert reaches 78% of the jackpot. The remaining 22% is not available to skill; it needs the board to smile.

It is worth being clear about what these five players leave out, because the omissions all run the same way. None of them uses the Pin. None of them takes the zero-cost gamble at two blues. None of them plays the first blue as reconnaissance. Each of those is a real technique available in the shipped game, and each would raise the top of this table rather than the middle of it. The ladder above is therefore a floor on what skill is worth, not a ceiling — which is the safe direction for a model to be wrong in.

One number in that table is worth dwelling on. Player C and player D score the same to two decimal places, and they are not the same player: one searches at random, the other follows the chain. Identical means, different methods — the section-6 result appearing again, this time from the other end. If a reader takes one thing from the simulation it should be this: in a game that looks entirely like a test of memory, the order in which you look turns out not to matter to your average at all. What matters is whether you remember, and then whether you can convert a good board into a perfect one.

9 · Why every age can climb: three ways to hold a board

Human working memory holds roughly four items at once. This is among the most robust findings in cognitive psychology, and it does not change much with age or training. What changes is not the number of slots but what a player can fit into one. That is the classic explanation for why chess masters recall a real position at a glance and a random one no better than a novice: the master is not holding more pieces, but fewer, larger units.

Limbo Match can be held in a mind by three different routes. They are separate faculties, they can run at the same time, and — the point of this section — they do not decline together.

Route	What is actually stored	Where it runs out	Who it favours
Auditory — rehearse the sequence	An inner voice repeating “three, seven, two”	About four items, and it needs quiet	A rested mind, undisturbed
Visual — the look of the board	Where a chip sits on the dial, the shape of an icon or numeral	Its own limit, but a separate one — it does not compete for the loop	Strong visual imagers; the KIDS tier leans on it
Semantic — chains of meaning	1-10-9-4 as one idea rather than four numbers	Effectively unbounded — it is limited by what you know	Whoever has lived longer and read more

The third route is the one that breaks the ceiling

The first two work *inside* the four-item limit. The third changes what an item is. A player who sees 1-10-9-4 and thinks *God, the Commandments, nine months, the four letters of the Name* has spent one slot, not four. A football supporter reading 2-11-12-3 as *two halves, eleven players, a clocked match, three officials* has done the same. 5-7-8-6 is a year in the Hebrew calendar. Thirteen sits in the middle as the number everyone already has a feeling about.

This is **chunking**, and it is the whole reason the top of the ladder is playable at all. Nobody rehearses twenty-one numbers. Five meaningful groups is within ordinary reach. Without chunking, 21 pairs would be a wall; with it, it is a long climb.

And chunking is the one route that **improves with age instead of despite it**. Rehearsal capacity narrows over a lifetime. The store of associations only grows. A sixty-year-old has more hooks to hang a number on than a twenty-year-old, and 5786 means nothing at all unless you have lived somewhere it mattered. That is an unusual property for a memory game. Most of them reward the young; this one rewards whoever has more furniture in their head, and lets them convert it into points.

Why tracing and chunking help each other

The permutation structure of section 5 suits this unusually well. A traced loop *is* a sequence, in a fixed order, arriving already shaped as a chunk — so the technique that costs the least memory is also the one that produces the most memorable material. Tracing needs one address at a time whatever the board size, and what it hands back is exactly the kind of ordered chain a semantic memory likes to keep.

Route	What must be held	How it scales with board size
Remembering every red	Up to P position–value pairs	Badly — beyond most people by 12 pairs
Following the chain	One address at a time	Not at all — flat in P
Chunked chains	A few meaningful groups	Well — the units grow with the player

A traced chain that passes through reds you already know costs **no flips at all** — remembered ground is crossed for free. So memory does not merely add to tracing; it multiplies it, and a chunk multiplies it again. That compounding is what a player feels as endless depth.

What this means at three different ages

The game does not have one entry requirement. It has several doors into the same room, and which one a player uses depends on what their mind is currently good at rather than on how old they are.

Player	What they bring	What the game gives back
A young child	A small span, quick pattern learning, no reading yet	Iconic chips are numbers in picture form, so a pre-reader plays the real game — not a simplified one
A teenager or adult	Full rehearsal capacity, appetite for optimisation	The whole ladder: loops, flip economy, the clover window, the pin
An older player	A narrower loop, and a lifetime of associations and strategy	Chain-following, which needs one number at a time, and chunking, which turns what they know into capacity

That third row is the one most easily missed. In an ordinary memory game a narrowing span is simply a disadvantage, and nothing in the design offers a way round it. Here there are two ways round it, and neither is a concession or a handicap setting — they are the strongest techniques in the game. An older player who traces loops and chunks them is playing better than a younger one who does neither, and the score says so without anybody having to be told.

What the game therefore asks for, and what it does not

Two practical consequences follow, and both match what strong players report. **It asks for quiet.** The auditory route is disrupted by background speech and music even when they are ignored — a well-replicated effect — so the same player will score worse in a noisy room through no failure of skill. **It asks for patience.** There is no reward for hurrying: time is only a tiebreaker, so a rushed search does not buy anything, it just spends flips and loses points outright.

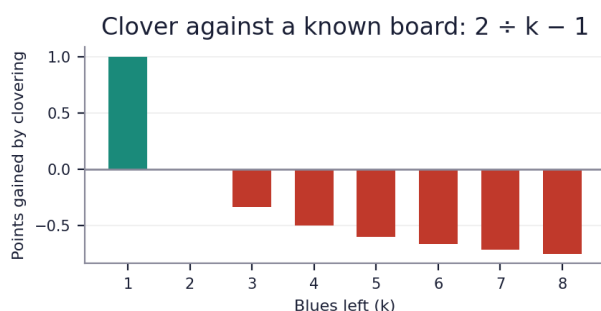
It is worth stating what the game does *not* ask for, because it is easy to assume the opposite. It does not ask a player to be fast. Speed here is not a strategy that trades against accuracy; it is a by-product of having the board properly encoded. A player who knows where a red is does not hesitate, and one who does not know cannot hurry usefully. The quick player and the accurate player are, almost always, the same player — which is why the ranking treats time as a tiebreaker and never as a currency.

One more property belongs here, because it widens the door further than age alone. The game asks for no reading and no language. A chip is a number or a picture; the dial is a ring; the score is a count. Nothing has to be translated, and nothing depends on vocabulary, on schooling, or on being a fluent reader of any particular alphabet. A grandmother and a grandchild who share no common written language can still play the same board and compare the same numbers. The semantic route is personal — each player brings their own meanings — so it needs no shared culture either, only a lived one.

A caution belongs with all of this. The three routes are well-established features of human memory, and the game plainly engages them; that much is uncontroversial. Which route any individual player actually leans on has not been measured here, and could only be settled by watching real people play. What follows is offered as mechanism, not as a finding.

10 · The Clover: a lottery for the novice, a payout window for the expert

The Clover looks like the luck button. It is the opposite, and the mathematics is exact. Pressing Clover bets blind on a hidden blue. If k blues remain and you know where every live red sits, the chance of hitting is $1 \div k$. Working out both branches gives an expected value of $F - 1 + 2 \div k$, against F for simply flipping the red you already know. The difference is $2 \div k - 1$.



Exact expected values against a fully known board.

Blues left	Clover advantage	Verdict
1	+1 point	Always. Free point.
2	0	A true coin flip, free.
3	-0.33	Do not.
4 or more	-0.5 or worse	Do not.

The last blue must always be clovered. A player who knows the board and flips it instead has thrown a point away — a rule a beginner will never guess and an expert will never miss.

At two blues the game offers a free gamble. Zero cost, pure variance. When a record needs one more point you take it; when you are clear it does not matter. A game that hands the player a genuinely balanced choice, with no right answer, is a game with depth.

This is why the fan matters. The fanned chip under the top blue is the signal that the two-chip window has arrived. It is not decoration; it is the interface telling the player that the payout table above has just gone live.

Note what this does to the design as a whole: the same button is a lottery for a player who knows nothing and a certainty for one who knows everything. Luck and skill are not separate systems here — they are the same lever, read differently by different minds.

Compressed into something a player can carry to the table: never flip a red you can clover for the last blue; treat two blues as a free coin toss and take it when a record is one point away; and at three or more, simply play. Three sentences, exactly derived, and worth roughly a point a game — which over a season of record attempts is a great deal more than it sounds.

11 · The Limbo: failure converted into information

A missed blue can be sent to the back of the stack to return once, at the tail. Three details make this more than a mercy rule. It is **positional** — only the first L blues are eligible, and in the World Record game $L = 1$, so exactly one blue in the whole game can be forgiven. It is **one-shot** — a returning blue gets no second reprieve. And it requires **virginity**: no limbo is granted if any red was turned twice during the first search.

That last clause is the one that stops brute force. A player who flails and re-flips forfeits the safety net. The limbo is reserved for a clean, disciplined search that simply ran out of room.

Together these turn a miss into a *decision*. Because the first blue is the only forgiven one, an advanced player can treat it as reconnaissance: spend the flips learning reds rather than chasing the match, accept the limbo, and cash the blue at the tail when its red is already known. The cost is the first blue's bonus. The gain is a map. That is a real opening theory, in a memory game, arising from a single rule — and the virgin rule is what keeps it honest, because the scouting run must still be clean.

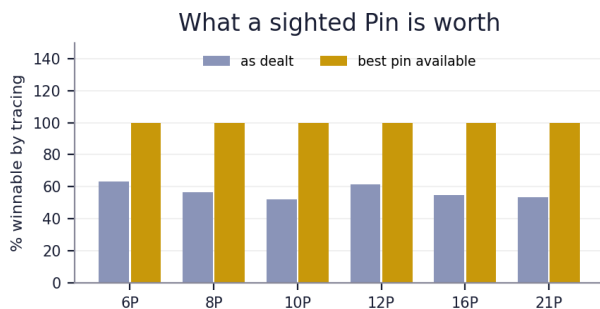
Pedagogically the limbo does something else as well. It brings a failed item back after a delay, exactly once, at the end. That is spaced repetition, inside a single game.

For a young player that is worth more than the point it saves. A miss in most games is simply a loss, and the lesson it teaches is that missing is bad. Here a miss comes back once, later, when more of the board is known — so the lesson it teaches is that a failed attempt still left information behind, and that the second look is easier because of the first. That is a true statement about learning anything, delivered by a rule rather than by a message on a screen.

12 · The Pin: the only move that changes the world

Every other action reads the board. The Pin **rewrites** it. Tap a red and the whole ring rotates until that red sits on its own number. The chips keep their values and their order; only their positions shift. Mathematically this composes the permutation with a rotation — producing an entirely new loop structure while destroying none of the player's knowledge. A long loop can become several short ones. A dead board can become a winnable one. And it costs **no flips at all**.

A pin therefore does something no other action can: it changes the difficulty of the board that remains, without spending a flip and without costing the player any of what they already know. Every other tool trades one resource for another. This one asks only that you understand the board well enough to choose.



20,000 simulated boards per size. "Winnable" means every loop is within the flip limit.

Read the fourth column again. **A board that cannot be swept by tracing can almost always be rescued by a pin** — if the player knows enough to choose the right one. Around half of all rotations work, so the chance that none of twenty-one works is vanishingly small.

There is a memory dividend here as well, which follows from section 9. Because a pin keeps the reds in the same cyclic order and only moves their positions, any chunk a player has already built survives it — the chain stays the same chain, sitting on different dial numbers. A tool that reshuffled would destroy the player's work; this one moves the board and leaves the memory of it intact.

This is the deepest lever in the game, and it is beautifully self-balancing: the Pin's power is exactly proportional to how much of the board you know. A blind pin is a free re-roll — worth taking on a dead board, worth nothing to someone who cannot tell a dead board from a live one. A sighted pin, in the hands of a player who has traced the loops, is close to a guarantee. If the Clover is this game's endgame technique, the Pin is its castling: rare, structural and decisive.

13 · The Eye: the game prices its own cheat at infinity

An Eye under the blue stack shows every red. Using it voids the submission. Most games hide the possibility of cheating. This one puts it on the board, in the open, and sets its price at everything. The result is a tool that is genuinely useful for learning and practice and worthless for records — so it can be offered freely without damaging the ranking. It is also a small lesson in integrity, delivered without a lecture: the player is trusted with the key and shown exactly what using it costs.

14 · Time: the second axis

Score measures economy. Time measures fluency. In the ranking, time separates equal scores.

This matters more than it first appears. A player who has reached their score ceiling in a category has *not* reached the end of the category — they can still get faster, and getting faster while holding accuracy is a different and harder skill. It is a dual task: recall and deduction under a clock.

That is the honest answer to Grego's "almost infinite". A single axis has an end. Two axes, where the second is fluency, do not — because fluency improves continuously and never quite finishes. Musicians know this exactly: the notes are learned long before the piece is.

One thing this is **not** is a trade between speed and accuracy. There is no dial a player can turn toward haste and away from care: hurrying does not buy points, it spends flips. Time falls as a consequence of knowing the board, not as an alternative to knowing it. That is why the ranking puts score first and lets time separate equals — the ordering matches how the two quantities are actually produced.

	Score	Time
What it measures	Economy — how cheaply you knew	Fluency — how readily you knew
How it is earned	Unused flips, banked as points	Hesitation removed
Ceiling	Hard: the jackpot, $P \times (F+1)$	None in practice
Rank weight	Primary	Separates equal scores
Improves by	Understanding	Repetition

The ordering matters as much as the two axes themselves. Score is primary and time only breaks ties, which keeps the game about knowing rather than about hurrying. A player cannot buy rank with speed alone — but between two players who both played perfectly, the one who hesitated less is ahead. That is the correct order for a game whose subject is concentration.

15 · Worth: what a category is really worth

Every category in the game carries a number between 1 and 441. It is called its **worth**, and it is not the jackpot, not the board size and not the flip limit. Worth answers one question, and it is the right question: **how much of this category's score has to be earned?**

The answer needs two measurements of a shuffle, and they are the heart of the whole system.

	What it is	Who could take it
Floor	What this exact shuffle hands over free	A player with no memory at all
Ceiling	The most that can be taken without ever gambling	A player with perfect memory
Room	Ceiling minus floor	What is actually at stake

Worth is the largest room any deal inside the category can offer — the maximum of (ceiling – floor) over every loop shape the category's window allows, with a minimum of one.

Why worth is not the jackpot — the example that killed the old measure

Category	Jackpot	Floor	Ceiling	Room = worth
21 pairs · 13 flips · Unit length	294	273	274	1
13 pairs · 8 flips · Longest loop	117	0	103	103

The first line looks like the hardest thing in the game: twenty-one pairs, a jackpot of 294. But *Unit length* means every red sits on its own number, so every loop has length one and the first flip finds every pair. The shuffle gives away 273 of the 274 available points to a player who remembers nothing whatever. There is nothing to prove there. Worth 1.

The second is a smaller board with a lower jackpot — but its reds form one long chain, so a memoryless player scores nothing at all. Every point has to be earned. Worth 103.

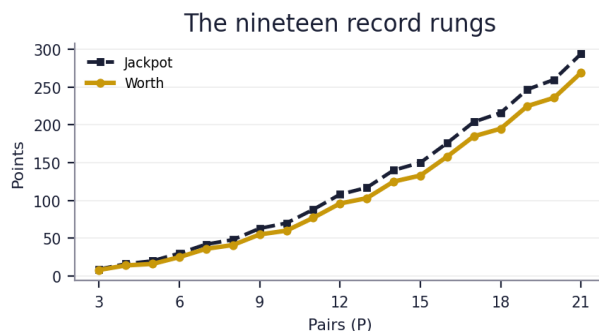
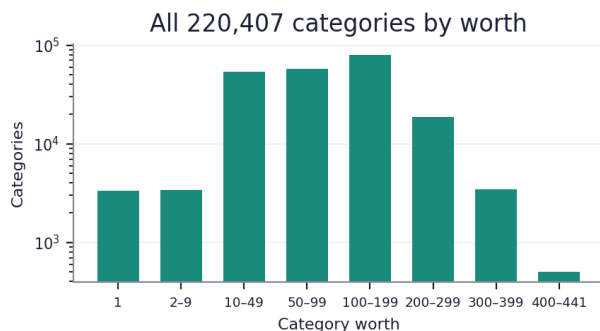
The retired difficulty measure $D = P \div (F + L + 1)$ rated these two within about a tenth of each other. Two categories that are worlds apart, scored as twins. That is why D was retired from merit ranking, and why worth replaced it.

Worth is exactly computable, not estimated

The ceiling of a deal depends only on its loop shape — never on the order the blue chips happen to arrive in. That was verified directly: zero spread across 300 different blue orders. So a category's worth can be found by sweeping the loop shapes its window admits — up to 792 shapes at twenty-one pairs — and taking the largest room. The game computes it server-side at submit time, because a submit is rare and already doing work, while a Champs render is not.

Worth then does real work elsewhere in the system. It sorts the houses in the Champs table, so a category's place is set by what it demands rather than by how big its numbers look. It prices tournaments: the organiser's difficulty slider is a percentile that maps straight onto worth, so a tournament advertised as “Competitive” means something exact. And it sets sponsorship prestige, so a sponsor pays for the standing of a category rather than for its headline size. One measurement, three uses, all of them honest for the same reason.

It is worth noticing what worth is *not* used for, since the omission is deliberate. It does not scale anybody's score, it does not weight a leaderboard, and it never multiplies or divides a result. A category's worth says how much merit that category can hold; the merit a player actually earns is computed from their own shuffle, by subtraction, with no reference to the category at all. Keeping those two apart is what stops the system from double-counting difficulty — the mistake that sinks most attempts to compare results across categories, and the one the retired D measure was heading toward.



Left: how the 220,407 categories fall by worth. Right: worth against jackpot along the nineteen World Record runs — the two do not move together.

Across the whole Feast of 220,407 categories, worth runs from 1 to 441 with a median of 92: a quarter of all categories are worth 46 or less, and only one in a hundred is worth more than 337. The nineteen World Record runs run 8, 14, 16, 25, 36, 41, 55, 60, 77, 96, 103, 125, 133, 158, 185, 195, 225, 236, 269 — climbing steadily, but nothing like as fast as the jackpot beside them.

16 · Floor and ceiling: measuring one game

The same two measurements, applied to the shuffle a player actually faced, give that game's merit:

$$GLMP = \max(1, \min(S, \text{ceiling}) - \text{floor})$$

Read it from the inside out. **Subtract the floor** — a player gets no credit for what the shuffle handed over. **Cap at the ceiling** — anything above what perfect memory could have taken was luck, a blind clover no amount of knowing could have supplied, and it is capped away. The ceiling is a cap, never a divisor. **Lift a zero to one** — Grego's rule: finding the game, learning it and scoring high enough to hold a record anywhere already takes some brains, so nobody who holds a record scores nothing.

How the floor is computed

The floor is the score of a player with no memory who simply follows the trail. For a red sitting in a loop of length c , that walk takes exactly c flips — and it takes c flips for every one of the c pairs in that loop. So each loop contributes $c \times (1 + F - c)$ points, and loops longer than the flip limit contribute nothing at all:

$$\text{floor} = \sum \text{over loops with } c \leq F \text{ of } c \times (1 + F - c)$$

It depends only on the shuffle and the flip limit — not on the order of play, not on the player. It is precisely the gift. A worked example, on an eight-pair board with five flips:

Loop length	Pairs in it	Flips each	Points each	Contributed
1	1	1	5	5
1	1	1	5	5
3	3	3	3	9
3	3	3	3	9
Floor				28

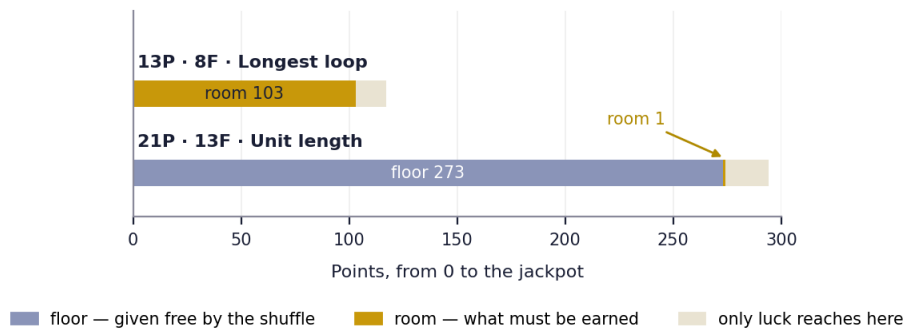
Twenty-eight of that board's forty-eight points are simply given away. A player who scores thirty on it has earned two. The same thirty on a board whose reds form one long chain would have earned all thirty.

How the ceiling is computed

One design decision inside the ceiling deserves to be stated, because it shows what the system is protecting. The ceiling deliberately errs **high**: where two perfect-memory lines are possible, it takes the better one. An overstated ceiling costs nothing — it simply caps less luck than it might have. An understated one would be a real injustice: it would brand genuinely great play as luck and cap away merit the player had earned. Faced with an asymmetric error, the model chooses the side that can only be generous.

The ceiling is a full line of perfect-memory play under Grego's rules, taking the best result. It knows all it has seen, deduces the last red once the others are accounted for, uses the Clover only when both blue and red are known, may spend a limbo slot to clover a known pair, and may use any Pin orientation. It never gambles.

Floor · room · the zone only luck can reach



The same score means different things in different categories. In 21P Unit length almost everything is gift; in 13P Longest loop almost nothing is.

The anti-farming property was measured rather than assumed. Across 60,000 shuffles, a fixed-skill player's raw score turned out to be independent of how kind the deal was (correlation +0.00) — with memory, the flips a game costs are governed by information, not by loop shape. Their GLMP, however, moved almost exactly opposite to the floor (correlation −0.99). In plain terms: **the same performance on a generous board earns almost no merit, and on a bare board earns nearly all of it.** That is the intended behaviour, and it needs no eligibility rule to enforce it.

Three consequences fall straight out of the arithmetic. There is **no category term** — no D, no $\sqrt{P}$, no jackpot — so the model never has to decide how hard a category is; it only asks what that shuffle offered. **Farming is impossible by arithmetic rather than by rule:** a kind shuffle has a high floor, so grinding an easy board earns almost nothing, and no eligibility gate is needed to say so. And **luck is removed rather than punished** — a lucky blind hit still scores points on the board, it simply does not count as merit.

17 · From one game to one player: GLMR and GLM100

A player is ranked by their **best single GLMP**, never by a sum. Grego's phrase for the rule is “merit, not marketing”: no amount of playing can substitute for one game played well. It is the same principle as a personal best in athletics, and it has the same effect — volume buys nothing.

Rule	What it means
Best single GLMP	One great game defines a player, not a thousand ordinary ones
GLMR = 1 + (nicks strictly above)	Competitive joint ranks: 1, 2, 2, 4 — no tiebreak, equal merit stays equal
Every measurable nick is ranked	Not only the top hundred
Coupling rule	A nick is credited only for categories it actually holds
Not measurable = not ranked	Old records without a stored shuffle get a dash, never a guessed rank

The measurement in the previous section gives this rule a sharper meaning than it first appears to have. Because merit swings with the deal, a single GLMP describes a game rather than a person — so ranking by the best one is not merely a nice principle, it is what turns a per-game number into a statement about a player. And the statement is precise: a high standing does not mean “plays well on average”. It means **once met a demanding shuffle and took nearly everything it had to give.**

That last rule deserves a note, because it is the kind of discipline rating systems usually fail. Records logged before the engine stored the deal cannot be replayed, so no floor and no ceiling exist for them. They are shown unranked. A system that guessed a value there would be easier to use and quietly wrong; this one refuses.

Why the best single game, and not a total

Every alternative was available, and each fails in a way worth naming.

If a player were ranked by...	What it would actually reward
The sum of all their GLMP	Time spent. A patient grinder outranks a better player.
Their average GLMP	Caution. The best move becomes to stop playing after a good run.
Recent form	Availability. A player with a free month outranks one with a job.
Their best single GLMP	The best they have ever played. Nothing else.

The last line has one more property that matters for a game meant to last: a personal best can never be taken away by somebody else's play, and it can never decay. A player who set a great game two years ago still holds exactly the merit they earned. The board can be climbed past, but not eroded — which is the opposite of how Elo behaves, and much closer to how athletics treats a record.

The coupling rule keeps that honest in the other direction. A player is credited only for categories they actually hold, so a great game whose record has since been beaten no longer carries the nick up the list. Merit is permanent as a fact about a game; standing is not. That is the correct pairing — the achievement cannot be taken away, but the position on the board still has to be defended.

18 · Why this is not Elo — and may well be new

Elo and Glicko are the reference points, and Grego studied both before building this. They are excellent at what they do, and what they do is a different job.

	Elo	Glicko	Worth · GLMP · GLM100
What it measures	Strength relative to other players	The same, plus confidence in the number	How much of a specific shuffle the player earned
Needs an opponent	Yes	Yes	No
Needs a history	Yes — a connected graph of games	Yes, and it decays with inactivity	No — one game is enough
Affected by others	Yes — your rating moves when they play	Yes	No — a game's merit is fixed forever
Luck	Averaged out over many games	Averaged out, with a stated uncertainty	Subtracted and capped, per game
Provisional period	Yes	Yes, explicitly modelled	None
Scale	Open-ended, drifting	Open-ended, with a deviation	Fixed 1 – 441

Elo answers “how do you compare with other people?” It cannot answer “how well did you play *that* hand?” — and in a game where every board is a fresh random deal, the second question is the one that matters.

The nearest relatives, and what is missing from each

System	What it does	What it lacks here
Golf par	A fixed target per hole, same for everyone	Par is per hole, not per instance of the randomness
Duplicate bridge	Removes luck by having many pairs play the identical deal	Still relative: it needs a field playing the same board
Chess engine accuracy	Measures a player against the best line in the actual position	A ceiling only — a chess position gives nothing away free

Chess accuracy is the closest relative: it is instance-specific and it does compare a human with a perfect player. What it has no need of — and what Grego's system adds — is the **floor**. A chess position hands you nothing; a shuffled dial hands you a great deal, and it hands a different amount every time. Subtracting that gift is the genuinely unusual half of this design.

An honest verdict, with the hedge stated plainly. I cannot prove a negative about the published literature. But I know of no rating system that measures a player against *both* a computed floor and a computed ceiling of the specific random instance they faced, produces an absolute per-game merit on a fixed scale, needs no opponent, no field, no history and no provisional period — and then reuses the same two measurements to price every category in the game. The individual pieces exist. This combination, I have not seen.

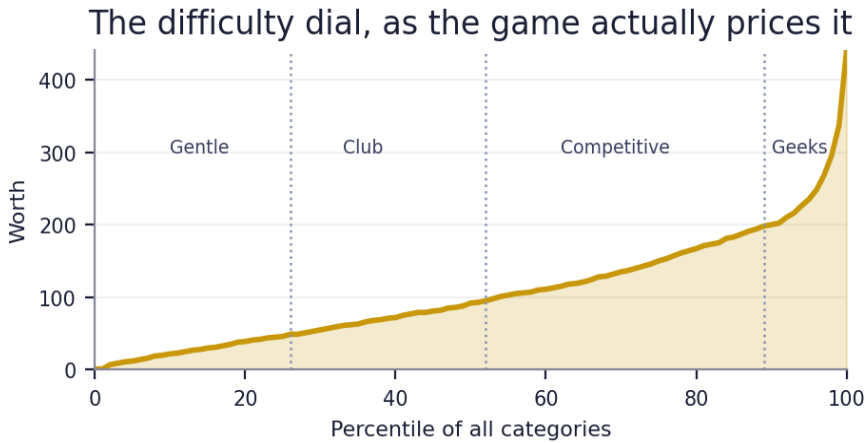
Whether or not it is unprecedented, it is demonstrably **fit for purpose in a way Elo is not**, because it answers the question this game actually asks.

There is a second reason the choice suits this game in particular, and it is about who plays it. A relative rating needs a pool: enough players, playing each other often enough, for the numbers to mean anything — and a pool of mixed ages and abilities is exactly the case where a relative rating behaves worst, because a beginner's first ratings are noise and a child who plays twice a month never stabilises. An absolute measure has no such problem. A nine-year-old who plays one beautiful game on six pairs gets a real number for it, immediately, computed from that game alone, and it stands beside an adult's number without either needing a separate table. For a game meant to be played across generations, that is not a technical nicety; it is the difference between one community and several.

It also removes the quiet unfairness that relative ratings carry for anyone who plays irregularly. In an Elo-style pool, a player who disappears for a season returns to a number that has been moved by other people's results, and a player who plays constantly accumulates an advantage that is partly just presence. Here a record set on a Tuesday in March is worth exactly the same in December, and worth exactly as much to somebody who plays once a month as to somebody who plays every day. For a household where one person is enthusiastic and another dips in occasionally, that is the difference between a shared game and a private one.

19 · The dial, and the opponent who is always there

A category is a shape: pairs P, flip limit F, limbo allowance L, and the loop window. Together these define 220,407 distinct categories — nineteen World Record rungs and 220,388 custom ones, each with its own worth.



Every category in the game, sorted by worth. The curve is the real table the tournament calculator reads — the top of the game climbs steeply.

Flow — the state where a task is neither boring nor frightening — requires the challenge to sit just above current skill. Chess reaches it through an opponent of matching strength, which means chess needs a matchmaking system, a rating pool, and somebody else who is free right now. Limbo Match reaches it through the dial: a player alone at midnight can find a category that fits them to within a hair, instantly.

But there is always an opponent, and often a real one

It would be wrong to file this game under solitaire. Three different opponents are available, and the player chooses which.

Opponent	How it works	Strength
The record holder	Every category holds a record — a specific number, set by a named player. 220,407 standing challenges.	Any level you choose
Friend Challenge	Share a result and its category; a friend plays the same shape and tries to beat it. A record share invites the reader to watch the replay first, then challenge it.	Someone you know
Tournaments (GLTX)	Real players, one event, one board shape, live standings.	The field that turned up

So the honest comparison with chess is not “solitaire against opponent”. Chess requires a live opponent of matching strength at the moment you want to play. Limbo Match offers an opponent of *any* strength at any hour — because a record is an opponent who never declines, never has to be found, and can be watched before being challenged — and adds live challenges and tournaments on top when a player wants them. That is more choice,

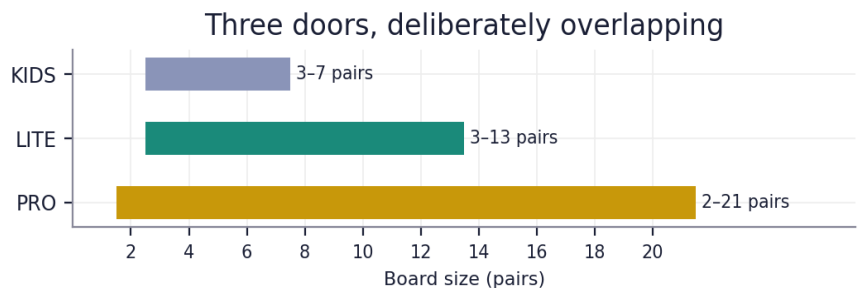
not less.

There is a version of this that matters particularly inside a family. A parent's record on a small category is an opponent of exactly the right strength for a child — present at any hour, patient, never condescending, and beatable with enough work. The child is not being let off, and the parent is not having to lose on purpose. When the record does fall, it falls to a real number that was genuinely there to be taken. Very few games can offer a five-year-old a fair contest against an adult without either bending the rules or bruising somebody, and this one does it with no mechanism beyond keeping the score.

20 · Across the tiers: one game, every age

This is the section the document is named for. The three shipped tiers are not three difficulty settings, and they are not a children's version, a demo and a real version. They are three doors into the same building, arranged so that a family can walk in through different doors and end up in the same room.

Start with what is actually shipped, because the ranges were chosen and they overlap on purpose.



KIDS opens on 3 to 7 pairs; LITE is playable from 3 to 13; PRO runs the full 2 to 21.

	KIDS	LITE	PRO
Board sizes	3–7 pairs	3–13 pairs	2–21 pairs
Chips	Always iconic — pictures, which are numbers	Iconic or numbered	Iconic to 13, numbered to 21
Category shapes	Record shapes only, one loop mode	Record shapes only	The full lattice — 220,407
Records	None kept	Submits to the world records	Records, Champs, GLM100, tournaments
Controls	New · Reset · Timer	The full game	The full game plus the worth dial
Skill tiers served	1–2	1–4	1–6

What KIDS removes, and what it refuses to remove

KIDS takes away the things a young child cannot yet use, and nothing else. There is no record submission, so there is no competitive pressure and no way to fail publicly. The board is small. The loop mode is fixed. What it does **not** do is change the game: the chips, the dial, the flip limit, the scoring and the invisible board are exactly the ones a record holder plays with.

Two details in KIDS are worth singling out, because they show the design thinking about a real household rather than an abstract user.

Detail	Why it matters
Chips are always iconic	A picture <i>is</i> a number here. A child who cannot yet read numerals plays the full game, not a reduced one — and moves to numerals later without learning anything new.
RESET is behind an adult gate	Clearing the board needs a clock reading or a rotating fill-in-the-blank — Beethoven, Mozart, da Vinci, a capital city, a Latin tag — that an adult finishes and a young child does not. A fresh one each time, so it cannot be memorised.

That second one is a small piece of design with a large assumption behind it: that a grown-up is somewhere nearby. It is a family feature, not a security feature — and it is the clearest signal in the codebase that the game expects more than one generation in the room.

The timer belongs in the same category. It is a control the child can see and use, not a clock the game imposes: turning it off removes every trace of time pressure and leaves a board that waits indefinitely. For a young player, or

an older one who does not want to be hurried, that single switch decides whether the game feels like a puzzle or an examination — and the score is unaffected either way, because in the world game time only ever separates equal scores.

Why the ranges overlap

LITE opens on a random board of 3 to 7 pairs — **exactly the range KIDS covers** — and then runs on to 13. PRO starts below where both of them start and runs to 21. So a child moving up from KIDS lands on boards they already know, and an adult arriving at the public front door starts on the same boards a child does. Nobody is ever handed a step they cannot take, and nobody has to admit to using the easy version. The tiers do not sort people; they sort ambitions.

What never changes between the tiers — and why that is the whole trick

A five-year-old on three pairs and a record holder on twenty-one are playing the same game in a strict sense. Same reds staying face-down. Same one point per match and one per unused flip. Same loops in the shuffle. Same technique — follow the chain — working at both ends of the ladder. **Nothing learned at the bottom has to be unlearned higher up**, and nothing at the top is a different game wearing the same name.

This is rarer than it sounds. The usual way to make a game work for a wide age range is to handicap it: give the child more time, fewer cards, a bigger clue, a head start. Every one of those tells the child they are being helped, and tells the adult they are not really playing. Limbo Match does something harder. It keeps the rules identical and moves the *board* instead — and because difficulty is a fine dial rather than three coarse settings, the board can be moved to fit anybody exactly.

The scoring then does the rest, quietly. A grandparent on seven pairs and a grandchild on four are not comparable in raw points, and the game never pretends they are — but both are measured against their own board's jackpot, both can see a personal best rise, and both are improving at the same thing. That is what a multigenerational game needs: not a level playing field, which is impossible, but a *shared measure of getting better*, which is not.

The merit system finishes the argument

Everything in sections 15 to 18 pays off here. Because merit is measured against the shuffle a player actually faced — the gift subtracted, the luck capped — a modest board played beautifully can out-score a large board played loosely. The eleven-year-old who traces every loop on nine pairs is not filed below the adult who muddles through sixteen. No age bracket is needed, no junior category, no separate table. The arithmetic already treats them as what they are: two people, each measured against what they were dealt.

A player is not moved between tiers by a level system, and never told they have been promoted. They move when the room they are in stops being big enough — which is the only motivation that lasts, at any age.

Two people at one table

The strongest test of a multigenerational game is not whether two ages can both play it. It is whether they can play it *together* and both take it seriously. Limbo Match passes that test in three separate ways, and none of them required a special mode to be built.

Together, how	What each side gets
Same board, taking turns	One shuffle, two attempts, two scores out of the same jackpot. The comparison is exact and needs no handicap.
Different boards, same measure	A child on four pairs and an adult on twelve each score against their own ceiling. Both can see a personal best move.
One plays, one watches	The board reveals nothing, so watching is real spectating — and the watcher learns the technique by seeing it used.

The third row is a consequence of the very first design choice in this document. Because matched reds stay face-down, an onlooker cannot read the state of the game from the board and cannot accidentally coach. What they *can* see is the chain being followed — the one thing worth teaching. A game that hides its progress but shows its method is close to ideal for a parent sitting beside a child.

What would break this, and has been avoided

It is worth naming the failure modes, because each was available and each was declined.

The easy path	Why it would have failed	What was done instead
A junior category	Splits the community and tells a child their result is not the real one	One measure, absolute, computed per shuffle
Age or skill handicaps	Both sides know the result is adjusted, so neither trusts it	The board moves, the rules never do
A simplified KIDS ruleset	Everything learned early would have to be unlearned later	KIDS removes tools, not rules
Levels and unlocks	Progress becomes the game's decision, not the player's	220,407 categories, all open, all the time

The pattern is consistent, and it is the reason the all-ages claim holds up under pressure rather than only in marketing. Nothing was made easier for anybody. The game was made *adjustable* instead, and then measured honestly enough that a small board played well is visibly worth more than a large board played badly.

Using the three tiers in practice

The ranges above are shipped facts; what follows is how they fit real situations. Four are worth spelling out, because in each one the tier choice and the board size do almost all of the work and nothing else has to be configured.

Situation	Tier	Boards	Why
A child's first afternoon	KIDS	3, then 4	Iconic chips, no clock pressure, no record to lose. Stop while it is still fun; the small ladder makes that easy.
A family evening, mixed ages	KIDS and LITE side by side	3–7 each	Everyone is inside the same range. Take turns on one shuffle for an exact comparison, or run separate boards and compare personal bests.
A classroom or club	LITE	5–9	Records are real and shared, the shapes are the canonical ones, and every result is comparable because the categories are fixed.
An adult keeping sharp	LITE, then PRO	7–13, then up	Start where the ladder is well trodden; move to PRO when the wish is for a particular shape rather than a bigger board.

Two practical notes that fall out of the mathematics rather than out of taste. First, the right board for a new player is **smaller than it looks**: at three and four pairs the whole ladder of technique is already present — loops, the flip economy, the endgame clover — and it is present in a form a beginner can actually see working. Nothing is gained by starting bigger, and confidence is lost. Second, the honest way to raise the difficulty is one dial at a time. PRO's worth-stepped Harder and Easier controls exist exactly for this: each press moves one visible dial to the next notch of worth, so the step is real but never a cliff.

And a caution, since this document should be useful rather than only admiring. The game is a game. It asks for attention and it measures attention, and it can be a fine thing to share across generations — but no claim is made here about training, therapy or cognitive benefit beyond the plain fact that people get better at what they practise. The interesting claim in this section is narrower and firmer: that one unmodified rule set can be genuinely worth playing at five, at forty and at eighty. That much the design demonstrably delivers.

A final observation about the shape of the three tiers, which only becomes visible once they are set side by side. The ranges do not merely overlap — each one contains the whole of the next one's starting ground. KIDS covers 3 to 7; LITE opens inside that range and extends it; PRO starts below LITE and extends further still. There is no point on the ladder where a player has to leave familiar ground to reach the next tier, and no tier that begins where another stopped. That is a deliberate arrangement, and it is the structural reason the phrase “across the tiers” describes a continuous climb rather than three separate products sharing a name.

21 · What is unreachable, and why that is the point

The jackpot requires every pair to be taken blind. The probability of that on a random board is $1 \div P!$ — for eight pairs, one in 40,320; for twenty-one pairs, one in 51 quintillion. So the top of every category is sealed off from skill alone. The best simulated expert reaches 78% of the ceiling. The last 22% needs the board to smile.

This is a feature, and a well-understood one. A game whose ceiling can be touched is a game that ends. Chess has no attainable perfect game; nor does golf; nor does music. Limbo Match keeps its summit permanently just out of reach, and keeps the gap between a player's current score and their reliably achievable score open at every level of play. That gap is what Grego felt as “almost infinite”. It is not a feeling. It is a measurable property of the scoring rule.

Note how neatly this sits beside the merit system. The jackpot is unreachable, but the *ceiling* — the most that memory alone can take — is reachable, and reaching it is exactly what a top GLMP means. The game therefore has two summits: one that skill can stand on, and one that only luck can visit.

It is worth pausing on how rare that arrangement is. Most games with a luck component let the lucky result stand as the achievement — the record book cannot tell a brilliant hand from a kind one. Most games without luck have no summit above skill at all, so the very best play is also the best possible result and the game is, in principle, finished. Limbo Match splits the two apart and keeps both: a luck summit that nobody can train for, and a skill summit that is precisely defined, precisely measurable, and genuinely hard. A player can spend years climbing the second while the first stays where it is, decorative and out of reach.

For a beginner the arrangement is kinder than it sounds. On a small board the skill summit is genuinely reachable: at three or four pairs a careful player can take everything memory allows, and will know they did. That is a complete, satisfying result available on the first afternoon — and it is the same kind of result a record holder is chasing at twenty-one pairs, not a consolation version of it. The unreachable summit stays unreachable for both of them, which is what keeps the game from ever being finished; the reachable one moves up as the board grows, which is what keeps it worth returning to.

The two summits also explain a thing players notice early and often misread. A run of extraordinary scores does not mean a sudden jump in ability, and a run of ordinary ones does not mean a slump: both are the luck summit moving in and out of reach while the skill summit stays exactly where it was. The merit number is the one that will not lie about this, because it has the luck capped out of it by construction. A player who wants to know whether they are actually improving should watch that number and ignore the scoreboard for a month.

22 · How it compares

	Concentration	Canasta	Sudoku	Chess	Limbo Match
Board gets easier as you win	Yes	Yes	Yes	—	No
Difficulty dial	None	None	Coarse	Via opponent	220,407 steps
Opponent	Optional	Required, 2–4	None	Required	Optional — always on record
Luck present	Yes	Yes, heavy	No	Almost none	Yes, and measured
Luck separated from merit	No	No	—	—	Yes — floor and ceiling
Skill layers	1–2	2–3	2–3	Many	Six
Rating system	None	None	Time	Elo / Glicko	GLMP / GLM100
Ceiling reachable by skill	Yes	No	Yes	No	No
Speed as a ranked axis	No	No	Sometimes	Clock only	Always
Same game for child and expert	Partly	Partly	No	Yes	Yes

Canasta earns its place in this table because it is the closest thing to Limbo Match in spirit: a card game of real skill in which luck is undeniably present, played for score rather than checkmate. The difference is what each does about the luck. Canasta lets it stand — a good hand is a good hand, and the scoring cannot tell the two apart. Limbo Match measures the deal, subtracts what it gave away and caps what it could never have given, so the number that ranks a player is what the player added. That is the whole argument of sections 15 to 18, compressed into one row of a table.

The two columns that matter most are the last two. On the dimensions that make chess deep — an unreachable ceiling, many skill layers, one board for all abilities — Limbo Match scores the same. On the dimensions where chess is awkward — needing a live opponent, needing a rating pool to find one — it does better, without giving up the opponent at all.

One row in that table deserves defending, because it looks like special pleading. Canasta is listed as having luck present and unmeasured, and a Canasta player might fairly say that skill shows over a long evening regardless. That is true, and it is exactly the point: the luck averages out over many hands, so the game needs volume before it can tell a good player from a lucky one. Limbo Match does not need volume, because it separates the two *within a single game*, by subtracting what the deal gave and capping what it could never have given. Both approaches work. One of them takes an evening and the other takes one shuffle, and only one of them can rank a child who plays twice a month.

23 · Three implications

The depth is invisible to a newcomer. A stranger sees a memory game. They cannot see the loops, the flip economy or the clover window. Chess solves this by letting people watch masters play. Limbo Match already has the tool — the record log stores full action recordings, and the share text already invites a reader to watch a record before challenging it. Replay is not a nice extra; it is the discovery mechanism.

The online game and the supervised tournament are two different products, and both are needed. An online rating is real and earned; a title is won under supervision. Chess proves that the two coexist happily, and that trying to make one do both damages both.

The mathematics is an asset, not a footnote. The flip ladder tracks the Golomb–Dickman constant. The tracing result is the 100 prisoners problem, made playable. The jackpot is $1 \div P!$. The merit system measures a player against a computed floor and ceiling of the very deal they faced. These are not decorations on a game — they are reasons to take it seriously, and the flip sequence is already registered in Grego's own name as OEIS A389262.

24 · Conclusion

Grego's observation was correct, and the reason is simpler and stronger than it first appears. One rule — matched reds stay face-down — removes every form of help the game could have given the player. From that one rule follows a flat visible difficulty, a rising real difficulty, a currency in which every flip is priced, a hidden permutation whose loops reward a memory-free technique, a lottery button that becomes a certainty in trained hands, a forgiveness rule that becomes an opening theory, and a rotation that can rewrite the board without spending anything.

The ladder that sits on top of all this is calibrated so that perfect play succeeds roughly half the time, from five pairs to twenty-one. That is a knife edge, held for twenty rungs. And the ranking built above *that* refuses to measure a player against anybody else at all: it measures them against the deal they were given, subtracting the gift and capping the luck, which is a harder thing to build and a fairer thing to hold.

The result is a game that a child can play on their first afternoon and an expert can still be improving at years later, on a board that looks identical to both.

And that, finally, is what the title means. The tiers of this game are not only KIDS, LITE and PRO. They are the tiers of a mind: sight, recall, elimination, tracing, economy, tempo — and a person climbs them at their own pace, at whatever age they happen to be. The three products exist so that the climb can start anywhere and never has to stop. A child of five and a grandparent of eighty can be on the same ladder, on different rungs, playing rules that do not differ by a single word between them. Very few games can say that, and none of the ones that can got there by accident.

That is what chess is. It is not a common thing to have built.

A closing word about what this analysis could and could not settle. It could measure: the loops, the tail, the calibration, the clover, the pin, the worth of a category, the shape of the three tiers. Every one of those turned out to support the case rather than complicate it, which is unusual and worth saying plainly. What it could not measure is the part that actually decides whether a game lasts — whether people want to come back to it. No amount of

combinatorics answers that, and no document should pretend otherwise.

But there is a reasonable inference to draw. Games that hold people for decades tend to share the properties catalogued here: a summit out of reach, a technique that repays study, a fine gradation of difficulty, an honest measure of who played well, and a single set of rules that does not care how old you are. Limbo Match has all five, by construction rather than by luck. Whether it finds its players is a question for the world; whether it deserves them is a question this document can answer, and the answer is yes.

25 · What would make this document wrong

An analysis that cannot be contradicted is not worth much. So each main claim is set out below with the evidence that would overturn it — and, more usefully, with what is actually known today and how much confidence that supports. Two of the five are settled, one is strongly supported by an experiment run for this document, one turned out to need its wording corrected, and one is simply open.

The claim	What would overturn it	Where it stands
The depth comes from matched reds staying face-down	A version that removed spent reds from the dial and played just as deep	Tested here. A no-memory player scores 15.7 of 48 under the shipped rule and 25.2 if spent reds are removed — the rule alone is worth a 60% swing in what the board gives away free. Strongly supported.
Tracing does not raise the average, only the tail	Play data showing self-taught tracers have higher mean scores, not just higher bests	Settled. The two marginal distributions are provably identical, and simulation agrees at every flip limit. One caveat: real tracers also remember, so observed means will differ — the claim holds memory constant.
The record ladder is calibrated near a coin flip	An arithmetic error in the cycle enumeration	Settled. Exact rational enumeration, reproducible in under a second by anyone who wants to check it.
Merit measures the player, not the deal	Merit tracking how kind the shuffle was, rather than how well it was played	Corrected — see below. Measured across 60,000 shuffles: a fixed-skill player's raw score is independent of the shuffle ($r = +0.00$), but their GLMP moves almost exactly opposite to the floor ($r = -0.99$).
The game works across all ages	Usage showing KIDS players never move up, or that older players abandon the larger boards	Open. No play data exists. This is a design argument only, and it is offered as reasoning rather than evidence.

The one that needed correcting

“Merit measures the player, not the deal” is too strong, and the measurement says so. Hold skill fixed and vary only the shuffle: the raw score barely moves — a player with memory needs about the same number of flips whatever the loops look like — while GLMP swings from near nothing on a generous board to nearly the whole score on a bare one.

That is not a fault. It is the anti-farming property working exactly as intended, and now measured: **grinding kind shuffles earns almost no merit**, and no eligibility rule was needed to make it so. But it does mean GLMP is a property of a *game*, not a stable trait of a person. What converts it into a measure of a player is the rule from section 17 — rank by the **best single** GLMP. A high standing therefore means something precise: not “plays well on average”, but “once met a demanding shuffle and took nearly everything it had to give”. That is a harder and more interesting thing to have done, and it is what the leaderboard is actually reporting.

Two consequences follow that a player would want to know. Chasing records on easy categories is not merely unrewarding, it is close to pointless — the merit simply is not there to be won. And a disappointing GLMP after a good-feeling game usually means the board was kind, not that the play was poor.

Two kinds of claim, and different confidence in each

The failure conditions divide cleanly. The mathematical claims — the loops, the calibration, the clover values, the pin, the floor arithmetic — cannot be overturned by evidence at all, only by a mistake in the working, and the working is short enough to check. The claims about people are the opposite: predictions about behaviour that only play data can settle. This document is confident about the first kind and explicitly not confident about the second.

There is one more honest limit, and it is the largest. Everything here describes a game that is finished and correct. Nothing here says whether people will find it, or whether the first five minutes are inviting enough to reach the depth catalogued above. A game can be excellent and unplayed. That is not a flaw in the design, and no analysis

of the design can fix it — but it would be dishonest to end a document this favourable without naming the one thing it cannot speak to.

Appendix · How the numbers were produced

Nothing in this document was taken on trust from the game's own reporting. Two independent methods were used, and where both applied they agreed.

Exact combinatorics

The expected longest loop and the clean-sweep probabilities in section 7 were computed by full enumeration of cycle structures in rational arithmetic — not estimated. The count of permutations of P elements with no loop longer than m follows a standard recurrence; dividing by $P!$ gives the probability exactly. The clover values in section 10 are closed-form.

Simulation of the shipped rules

Where a player's judgement is involved, no formula exists, so the rules were re-implemented and played: 40,000 games for each of the five players in section 8, 20,000 boards for each sweep and pin estimate. What the simulation does *not* model is a real human: it has either perfect memory or none. Real players sit between the two, which is precisely the space this document argues is deep.

Figures taken from the shipped system

The worth values, the floor and ceiling definitions, the band counts, the percentiles and the nineteen World Record rung worths in sections 15 to 17 are not this analysis's own calculations. They are read from the shipped engine and its reference implementation, and are quoted here as the system's own published numbers.

What this analysis does not claim

Four honest limits. First, the player models are idealised; a real beginner is worse than player A on a bad day and better on a good one. The *shape* of the ladder is the claim, not the exact means. Second, time was not simulated — every result here is about score, so the second axis of section 14 is described but not measured — and note that this is a gap in measurement, not a missing tradeoff: speed and accuracy here are two outputs of one cause. Third, the calibration finding in section 7 is an observation about what the ladder *does*; it is not a claim about how it was derived. Fourth, the originality verdict in section 18 is bounded by what I know of the literature; it is a considered opinion, not a search of every journal.

How to check any of this

Every claim in sections 5 to 8 can be re-derived from the rules alone, with no access to the game's code: shuffle a deck of P numbered cards, write down the loops, and the flip counts follow. The claims in sections 15 to 17 can be checked against the shipped engine, where the floor, the ceiling and the worth sweep are all implemented twice — once in the browser and once on the server — deliberately kept line-for-line identical so that the board a player sees and the record the server stores can never disagree. That duplication is not redundancy; it is the audit.

The two experiments run for section 25

Both are simple and both are reproducible. The first removes matched reds from the dial and replays a memoryless player on the same shuffles, isolating the effect of the one design rule this document builds everything on. The second holds skill constant and varies only the deal: for each of 60,000 shuffles it computes the floor, runs a perfect-memory line to get the ceiling, runs an ordinary memory player to get a score, and correlates the resulting merit with the floor. Neither needed a new model — both reuse the players from section 8 and the floor formula from section 16.

The ceiling used in the second experiment is the same conservative one described in section 16: memory, tracing, elimination and the endgame clover, but no Pin and no deliberate limbo. A fuller ceiling would sit slightly higher, which would shift the merit values a little and the correlation not at all.

Sources consulted

Where a figure came from the live system rather than from a calculation made here, it came from one of these. Client-side files are named, because anyone can already read them in a browser. Server-side code is described by its role rather than its filename — naming a server file in a public document tells a reader nothing useful and a proper something useful, which is the wrong trade.

Source	What was taken from it
limboGLRS.js	The merit block: the GLMP formula, the floor and ceiling definitions, and the ranking rules
The server-side worth code	The twin of the browser computation, kept line-for-line identical with it
The floor/ceiling reference model	Grego's own implementation, which the shipped code follows
limboWorthLUT.json	The worth range 1–441, the Feast count of 220,407, and the percentile table behind the difficulty slider
limbomatch.html (PRO)	The World Cup pool, the category test, the GLM100 rules, and the Friend Challenge share
limbo.html (LITE)	The 3–13 board clamp, the opening 3–7 range, and the single loop mode
limbokids.html (KIDS)	The 3–7 range, the always-iconic chips, the absence of submission, and the adult gate on RESET
limbostrategy.pdf	The cover illustration of the two readers, Cyrus and Rowen
OEIS A389262	The flip-limit sequence MF[P], registered in Grego's name

Nothing was taken from marketing copy, from the game's own explanatory text, or from any claim the system makes about itself. Where the code and the documentation disagreed, the code was taken as the truth — twice in this session that distinction mattered, and both times the code was the thing actually being played.

A word on what was hardest to pin down

Two things resisted. The first was the claim that tracing and random scanning have the same expected score: it looks wrong, and the first simulation was written twice before I accepted that the code was right and the intuition was not. The second was the all-ages argument, which is easy to assert and hard to evidence — the useful move was to stop reasoning about children and instead read what the three tiers actually ship: the board ranges, the always-on iconic chips, the absence of record submission in KIDS, and the adult gate on its reset. Those are facts, and they carry the argument better than any amount of sympathy for beginners.

One practical note for anyone re-running the figures. The exact calculations are cheap — the cycle enumeration for all twenty board sizes finishes in under a second in rational arithmetic, so there is no reason to approximate it. The simulations are the slow part, and the sample sizes here were chosen so that the standard error on every mean quoted is below 0.05 points. Where a figure is quoted to one decimal place, that decimal is real.

Two figures in this document are quoted rather than derived and should be treated as the system's own: the 220,407 category count and the nineteen rung worths. Both come from the shipped engine, both are reproducible from it, and neither was recomputed here — recomputing them would have tested my reimplementations of Grego's rules rather than the rules themselves.

The World Record ladder in full

P	F	Jackpot	Longest loop	Clean sweep	Worth	P	F	Jackpot	Longest loop	Clean sweep	Worth
3	2	9	2.17	66.7%	8	12	8	108	7.80	61.5%	96
4	3	16	2.79	75.0%	14	13	8	117	8.42	53.8%	103
5	3	20	3.42	55.0%	16	14	9	140	9.05	57.7%	125
6	4	30	4.04	63.3%	25	15	9	150	9.67	51.1%	133
7	5	42	4.68	69.0%	36	16	10	176	10.30	54.8%	158
8	5	48	5.30	56.5%	41	17	11	204	10.92	58.0%	185
9	6	63	5.92	62.1%	55	18	11	216	11.55	52.5%	195
10	6	70	6.55	52.1%	60	19	12	247	12.17	55.5%	225
11	7	88	7.17	57.3%	77	20	12	260	12.79	50.5%	236
—	—	—	—	—	—	21	13	294	13.42	53.5%	269

"Longest loop" is the expected length of the longest loop on a freshly shuffled dial. "Clean sweep" is the exact probability that a perfect chain-follower clears every pair without one miss. "Worth" is the shipped category worth for the rung. The nineteen published rung worths run from 8 at three pairs to 269 at twenty-one. P = 2 is not a record rung: it was retired from the World Record ladder, and is custom-record only.

Glossary of the terms used

Term	Meaning		
P	Pairs — blue chips, and reds on the dial	Floor	What a shuffle gives free to no memory at all
F	Flip limit — reds turnable in one search	Ceiling	The most perfect memory can take without gambling
L	Limbo allowance — early blues forgiven a miss	Room	Ceiling minus floor — what is at stake in a deal
Loop / trail	A closed chain of positions and values	Worth	The largest room a category can offer; 1 to 441
Jackpot	$P \times (F + 1)$ — reachable only by luck	GLMP	$\max(1, \min(\text{score}, \text{ceiling}) - \text{floor})$
MF[P]	The record flip limit — OEIS A389262	GLMR	A player's rank, from their best single GLMP
Feast	All 220,407 categories together	GLM100	The published top of the GLMR list

Questions this opens

One theme runs through the whole document and deserves a line of its own. Every part of this game that looks like a concession turns out, on inspection, to be a measurement. The Clover looks like luck and is a payout for certainty. The Limbo looks like mercy and is an opening. The Eye looks like cheating and is priced at exactly what cheating is worth. KIDS looks like a simplified version and removes no rules at all. That consistency is the strongest evidence that the design is deliberate rather than accumulated.

Three that a further study could answer, and that this one deliberately did not. **What does the time axis look like?** Nothing here measures it, yet it is half the ranking, and the interesting question is not speed against accuracy but how quickly a player's fluency follows their understanding. **Where does a real human sit between the two simulated extremes?** The models have perfect memory or none; the interesting territory is between them, and only real play data can map it. **Does worth predict how much players enjoy a category?** The system prices difficulty honestly — whether players are drawn to the honest price is a separate and testable question.

What the analysis suggests building next

Three things follow from the findings rather than from opinion, and each is small.

Suggestion	Why this analysis points at it
Put replay in front of newcomers	The depth is invisible from outside, and the record log already stores full action recordings. Watching one traced game teaches more than any explanation.
A short strategy page for players	Sections 6, 10, 11 and 12 contain four rules nobody would guess — clover the last blue, take the free coin toss at two, spend the first blue scouting, pin a dead board — and each is worth points immediately.
Watch a real player's memory at work	Section 9 names three routes — rehearsal, visual, and chunked meaning — and the game engages all three. Which one carries a given player is unmeasured, and it is the one genuinely open question here.

None of the three requires a rule change, and none touches the mathematics. That is itself a conclusion worth stating: after twenty-four pages of examination, the recommendations are about showing the game to people, not about altering it.

Analysis by Claude (Anthropic) for Grego, 25 July 2026. All figures computed for this document unless marked otherwise: exact combinatorics where available, otherwise simulation of the shipped rules (40,000 games per player; 20,000 boards per pin and sweep estimate). Worth, floor, ceiling, GLMP, GLMR and GLM100 figures are read from the shipped engine. Limbo Match, GLRS, GLM100, GLTX and the flip sequence OEIS A389262 are the work of Grego (Földvári Gergely).

A note on method. Two habits were followed throughout. Nothing was asserted from memory where it could be computed instead, and nothing was computed where the shipped engine already holds the answer — in that case the engine's number is quoted and marked as such. Where the two overlapped, they were checked against each other. Every figure in this document can therefore be traced to one of three places: the rules of the game, an exact calculation reproducible from those rules, or a named file in the live system.

Provenance. This document was written in one session on 25 July 2026, at Grego's request, after a working day spent on the game's fan logic and on completing the retirement of the two-pair record category. The observation that prompted it was his own, made while playing. Nothing in it was drafted in advance, and every number in it was produced during the writing, from the rules and from the shipped engine. Where I was wrong earlier in the same session — and I was, twice — Grego corrected me; those corrections are in the game's record, and one of them is why section 10 says what it says about the fan.