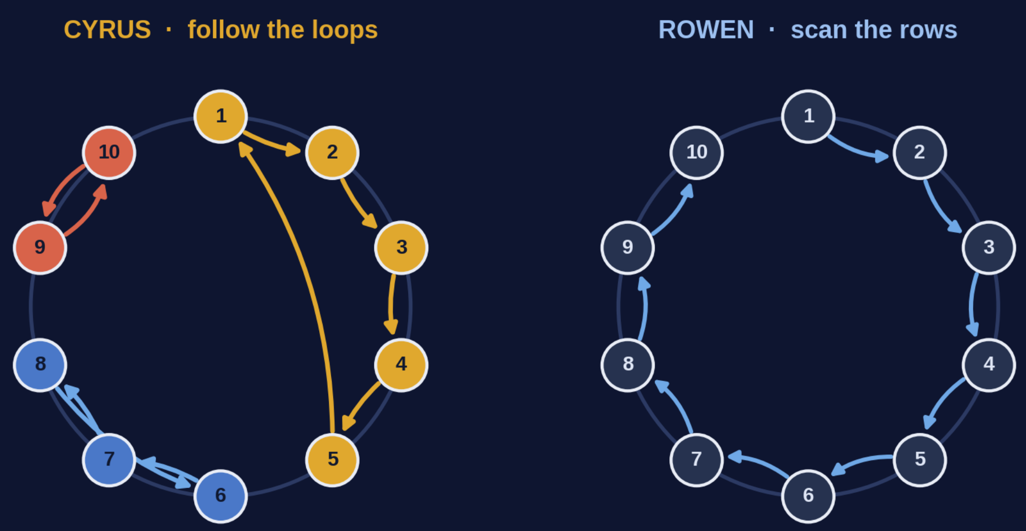
GREGO'S LIMBO MATCH™

MEMORY · STRATEGY · LOTTERY

The Two Readers

Cyrus the cycler, Rowen the rower — strategy, simulation, and cognitive load across the Feast



Two readings of one Shuffle. Cyrus follows the loops — each chip he turns names the next chip to turn. Rowen scans the rows — positions in plain order. P = 10, loops {5, 3, 2}.

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grego.hu/limbo · OEIS A389262 (CST) · 21 May 2026

THE QUESTION

Two readers of one Shuffle

A Shuffle hides P red chips in a secret order. Two players with perfect memory read it two different ways. Does the way they read change the score — and should the ranking care?

Cyrus reads by *loops*. He flips the chip whose position equals the value he is hunting, reads what it shows, and flips that next — the chain leads itself, because in a Shuffle a position points to a value and that value names the next position. **Rowen** reads by *rows*. He turns the dial positions in plain order, 1, 2, 3, …, banking what he sees until the chip he needs turns up. Both remember everything; every flip costs time, and time enters the score. The arena is the **World Record line** — one category per P from 3 to 21, all **MIXED** (any loop length), **L = 1** limbo, and flip limit **F = CST**, the Critical Survival Threshold A389262(P).

This paper settles the contest in three steps: a clean **tie** under perfect sighted play; a decisive **break** once the full toolset is in play; and a **field study** of how the gap moves across the Custom field. It closes with the human floor — what each style demands of a real memory as P grows — and what all of it means for the GLRS ranking.

THE GAME

Mechanics, in brief

A category fixes four dials: **P** pairs (2–21), **F** flip limit (1–P), **L** limbo come-backs (0–P), and the loop range **LL = [MIN, MAX]**. Flip a blue chip, search its red match inside F flips, or *blind-guess* with the clover and bank the unused flips as bonus. The score ceiling per category is the **Jackpot**:

$$\text{Jackpot} = P(F+1) \qquad \text{per category}$$

A missed blue is not lost: it drops to the bottom of the stack and resurfaces **last** — the **limbo** — with a far better second chance. One rule guards it: the **virgin rule** — flip a red twice in the first search and the limbo is void. The **World Record (WR)** line is the calibrated midline (MIXED, L = 1, F = CST); the **Custom Records (CR)** open the whole field — the **Feast of 220,407** categories (the 19 WR plus 220,388 CR), counted and weighed in *The Maths of the Feast*. One structural fact drives everything below: at the CST, **F > P/2 for every P**, so a Shuffle can hold at most one loop longer than F.

P	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
F = CST	2	2	3	3	4	5	5	6	6	7	8	8	9	9	10	11	11	12	12	13

The WR flip limit per P — the CST sequence A389262, by Grego (Gergely Földvári).

THE FLOOR

Pure sighted play is a tie

Strip the game to plain matching — no clover, no tactics, both players sighted and flawless — and the two readers are **indistinguishable**. The reason is structural. Because F > P/2, a Shuffle holds **at most one loop longer than F**, so a sighted player suffers **at most one forced miss** per game — and the single L = 1 limbo always recovers it. Both therefore match all P pairs, every time. The contest collapses to a **flip race**:

$$\text{flips}_{\text{Cyrus}} = 2P - m\,(\#\text{loops}) \qquad \text{flips}_{\text{Rowen}} = 2P - r\,(\#\text{records})$$

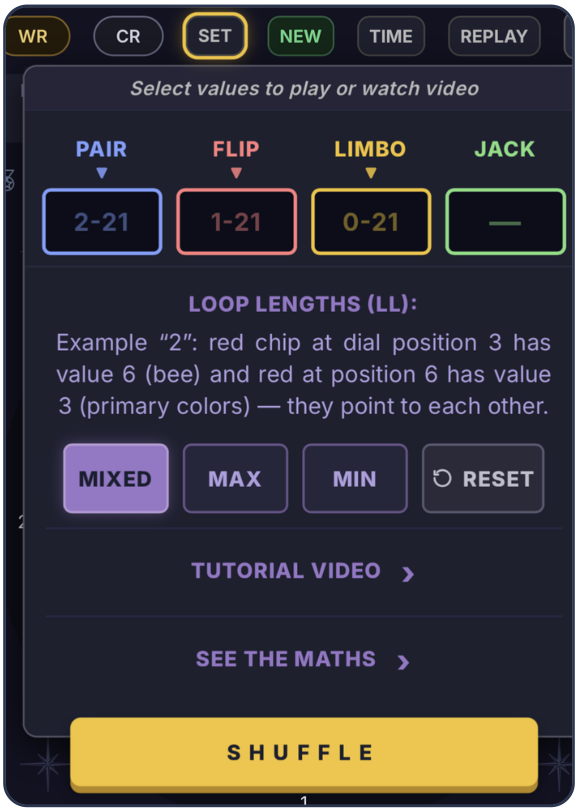
A loop is closed for free the moment its last unknown is forced (a record, for Rowen; a returning chain, for Cyrus). By the Foata–Schützenberger bijection the number of loops and the number of left-to-right records share one distribution, with the same mean — the harmonic number:

$$\mathbb{E}[\,m\,] = \mathbb{E}[\,r\,] = H_P = \sum_{k=1}^{P} \frac{1}{k} \approx \ln P + \gamma$$

So their expected flips are **equal, category by category**. A Monte-Carlo party over all 19 WR categories (N = 80,000 each) confirms it to the decimal:

> **Cyrus 1961.50 · Rowen 1961.45 · difference +0.05.** The 95% confidence interval straddles zero; wins split 49.2% Cyrus / 48.9% Rowen. A coin-flip.

An athletes' relief — the flip race of Result A in stone: under flawless sighted play, the two readers finish level.



The live category picker on grego.hu — PAIR (P), FLIP (F) and LIMBO (L) set the three numeric dials; the loop window LL is chosen with MIXED / MAX / MIN. The worked example is a 2-loop: position 3 → value 6, position 6 → value 3 — the self-pointing chain Cyrus follows.

The table below is the exact engine of the tie: for each WR rung, the expected loop count equals the expected record count (H_P), so the expected flips (2P − H_P) match. The Jackpot is the per-category ceiling; the last column previews the full-game edge of the next section.

P	F = CST	Jackpot P(F+1)	E[loops]=E[records] = H_P	E[flips] = $2P - H_P$	Cyrus edge (full game)
3	2	9	1.83	4.17	+1.33
4	3	16	2.08	5.92	+1.99
5	3	20	2.28	7.72	+1.87
6	4	30	2.45	9.55	+2.40
7	5	42	2.59	11.41	+2.99
8	5	48	2.72	13.28	+2.83
9	6	63	2.83	15.17	+3.43
10	6	70	2.93	17.07	+3.33
11	7	88	3.02	18.98	+3.87
12	8	108	3.10	20.90	+4.47
13	8	117	3.18	22.82	+4.34
14	9	140	3.25	24.75	+4.93
15	9	150	3.32	26.68	+4.89
16	10	176	3.38	28.62	+5.36
17	11	204	3.44	30.56	+5.95
18	11	216	3.50	32.50	+5.89
19	12	247	3.55	34.45	+6.46
20	12	260	3.60	36.40	+6.45
21	13	294	3.65	38.35	+6.96
Σ	—	2298	—	—	+79.7

Derived quantities computed exactly; the edge column is the measured full-game advantage (Result B). Jackpot sum 2298 is the party ceiling.

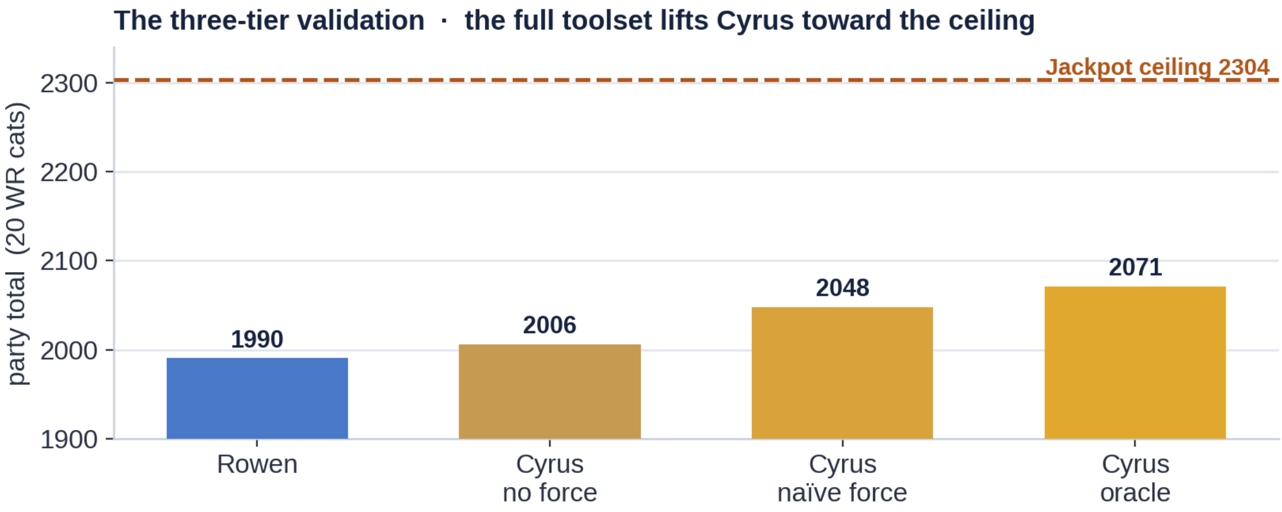
Reading. The loop range adds *no intrinsic difficulty* on its own — perfect play ties regardless of how the Shuffle breaks into loops. Whatever separates the readers must come from the *tools*, not the bare match.

THE BREAK

The full game tips to Cyrus

Three tools live above bare matching. The **clover** pays the maximum bonus (F + 1) for a blind match — better than any sighted search. The **limbo** returns a missed blue last, when the field is nearly known. And the **fanned tail**: as the stack empties, the surviving chips fan out, exposing by elimination what each must be.

The decisive move is **force-limbo**, and only Cyrus can play it. Knowing the loops, he can *decline* the match on the longest loop's first blue — a clean miss, no second flip, so the virgin rule holds — and bank that blue into the limbo. Its long trace then converts into **two** tail bingos: the forced blue itself, and the fan-lit penultimate chip deduced once the fan reveals the limbo's value. Rowen scans blind; he cannot decline a match he has not yet located, so he cannot force. The result is a clean, growing separation:

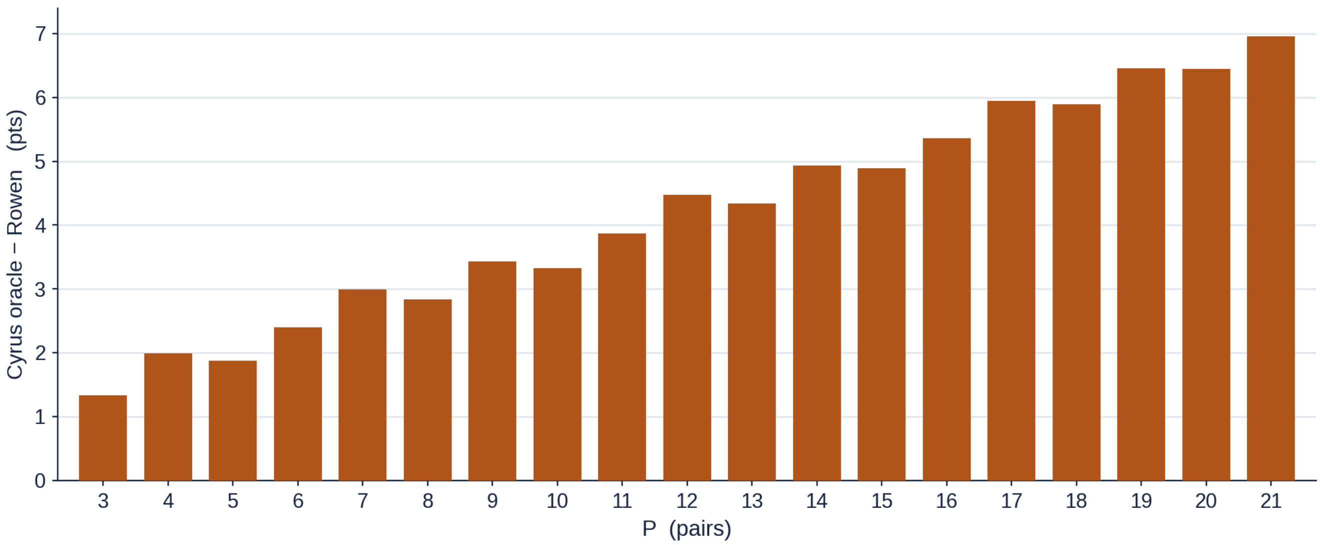


Party totals over the 19 WR categories (N = 40,000 each). Each tool Cyrus adds lifts him toward the Jackpot ceiling of 2298; Rowen sits at the sighted floor.

Player	Tools used	Party total	vs Rowen
Rowen	sighted + clover + limbo	1985.9	—
Cyrus — no force	+ cycle-lit fan	2001.7	+15.8
Cyrus — naïve force	+ force-limbo	2042.6	+56.7
Cyrus — oracle force	+ partition-aware stop	2065.6	+79.7
Jackpot ceiling	**—**	**2298**	**—**

The three-tier validation: no-force, naïve-force, oracle-force, each measured against Rowen on identical Shuffles.

The full edge decomposes cleanly. Cycle-knowledge alone — just lighting the fan correctly — is worth **+15.8**. The force-limbo tactic adds **+63.9**, of which **+23.0** is the premium for choosing *which* loop to force well (oracle over naïve). And the advantage is not flat: it climbs steadily with P, from about +1.3 at P = 3 to +7.0 at P = 21.



Per-category advantage (Cyrus oracle − Rowen) by P. The edge rises almost linearly in P — more loops mean more structure for Cyrus to exploit.

THE TACTIC

Optimum stopping into the Feast

Force-limbo poses a real decision: *which* loop should carry the single limbo? Cyrus wants the **longest** loop — its trace seeds the richest tail — but he meets the loops one at a time as he traces, and must commit before he has seen them all. This is an **optimal-stopping problem**, a secretary problem played on cycle lengths, with one twist: if he waits too long, the natural forced miss (the loop beyond F) lands itself on whatever loop happens to overflow, possibly a short one. So the policy carries an **overflow hedge** — commit early enough to steer the limbo, late enough to catch a long loop.

The fuel for that policy is the loop-length distribution — exactly the partition mathematics that sizes the Feast. Knowing how a random Shuffle of P tends to break (the cycle-type weights summing to P!) lets Cyrus set his stopping threshold optimally. That knowledge is the **+23.0** premium of oracle over naïve forcing: the same partition counts that price the categories in *The Maths of the Feast* also price this tactic.

$$\text{Feast} = \sum_{P=2}^{21} C_P \cdot P \cdot (P+1) = 233{,}380$$

THE FIELD

How the edge moves across CR

Beyond the WR midline, the Custom field exposes which dials the reader's edge actually rides on. Sweeping one dial at a time (anchored at P = 12) tells a consistent story: **Rowen barely moves** — he plays the sighted floor in every setting — while **Cyrus's score does the travelling**.

The loop window LL

Window	LL	Rowen ≈	Cyrus ≈	edge	edge %
Unit	[1, 1]	91.4	96.9	+5.5	6.1%
≤ Flip	[1, 8]	91.4	94.8	+3.4	3.7%
MIXED	[1, 12]	91.4	95.9	+4.5	4.9%
5-MIN	[5, 12]	91.4	96.5	+5.1	5.6%
Longest	[12, 12]	91.4	97.9	+6.5	7.2%

P = 12, F = 8, L = 1. The MIXED tie of the WR line is a knife-edge; constrain the window toward long loops and Cyrus pulls clear.

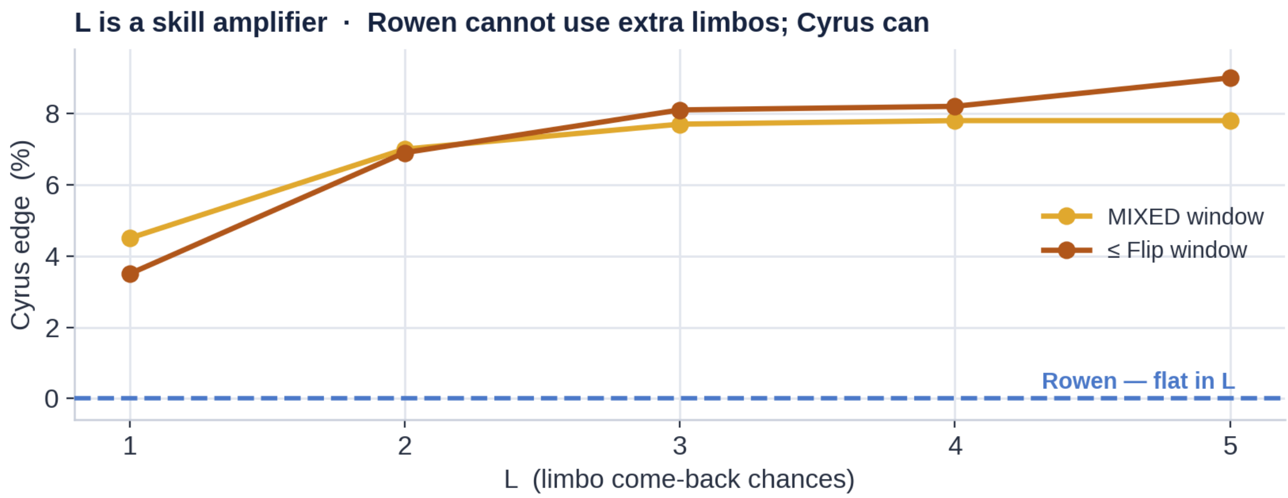
The flip limit F

F (MIXED)	edge	edge %
6	+3.4	4.9%
8	+4.5	4.9%
11	+6.9	5.5%

P = 12, L = 1. A larger flip budget gives Cyrus more room to convert structure into bonus; the edge grows with F.

The limbo count L — the sharpest axis

L is where the readers diverge most. With F > P/2 a sighted player has at most one natural miss, so **Rowen cannot use extra limbo slots** — his score is dead flat in L. Cyrus, who *forces* misses on purpose, fills them: each extra L lets him force another loop and deepen the fan tail. The edge roughly **doubles by L = 2** and plateaus near L ≈ 3, about the loop count H_P.



Cyrus edge versus L. Rowen is flat along the dashed line; Cyrus climbs until he runs out of loops to force.

L	MIXED edge (pts)	≤ Flip edge (%)
1	+4.5	3.5
2	+7.0	6.9
3	+7.7	8.1
4	+7.8	8.2
5	+7.8	9.0

L is a pure skill amplifier for the loop-reader; F and the long-loop side of LL are the durable edge axes.

THE LOAD

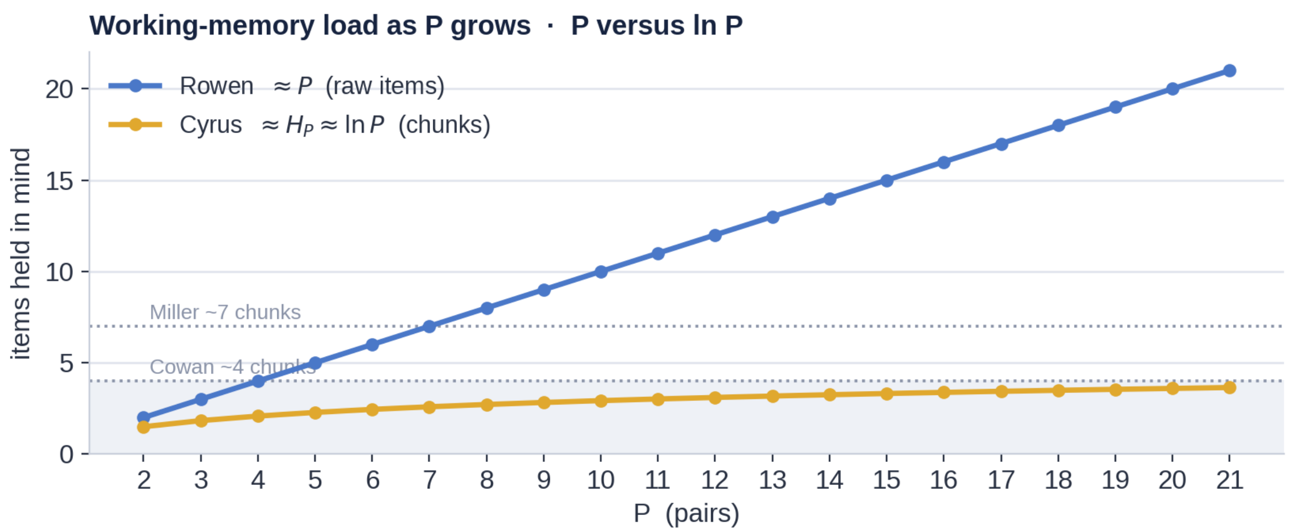
Memory, as P grows

The perfect-memory tie of Result A is the **floor** — the best possible case for Rowen, with his memory burden waved away for free. No human meets that assumption. Ask instead what each style actually demands of a real head.

Rowen carries an unstructured table. Row-scanning offers no organizing principle: every red he turns is an arbitrary (position → value) fact he must bank, because any of them may answer a later blue. To find blue v he scans that mental table for where value v appeared — associative recall over a flat, structureless set that grows to size P. His load is **O(P) raw items**, like memorizing a random P-digit string.

Cyrus offloads onto the board. Cycle-following is *self-cueing*: the value he just turned over names the next position to flip. During a trace he need not remember where to go — the chip points there. Tracing one loop costs **O(1) working memory regardless of its length**; he holds only the start and the current link. What he tracks is just the *set of loops*, and a random Shuffle has only **m ≈ H_P ≈ ln P** of them — each a tidy, self-consistent chain held as a single chunk.

$$\text{load}_{\text{Rowen}} = \mathcal{O}(P) \qquad \text{load}_{\text{Cyrus}} = \mathcal{O}(H_P) = \mathcal{O}(\ln P)$$



Working-memory load against P. Rowen's raw-item count climbs past both the Cowan (~4) and Miller (~7) chunk ceilings; Cyrus stays inside the shaded band for every P the game allows.

P	Rowen items (≈ P)	Cyrus chunks (≈ H_P)	vs ceiling
4	4	2.1	Rowen at Cowan ~4
7	7	2.6	Rowen at Miller ~7
10	10	2.9	Rowen ~2.5× over Cowan
21	21	3.6	Rowen ~5× over Cowan

Human working memory holds ~4 chunks (Cowan) to ~7 (Miller). Rowen breaches the ceiling by P ≈ 4–7; Cyrus stays under it for all P (H_P reaches 4 only near P ≈ 31).

The overload is not free — it couples straight into the score. A forgetful Rowen **re-flips** reds he has already seen: wasted flips (time, lost bonus), and worse, re-flipping a red in the first search **voids the limbo** by the virgin rule, costing whole pairs. Cyrus's self-cued trace almost never re-flips, so his limbo and bonuses survive. So the practical gap is ~0 at small P and **widens with P** — the same direction as the tool-driven edge. The two effects point the same way and compound.

THE VERDICT

What the ranking should do

The findings converge on the **worth model**. Result A shows the loop range sets no intrinsic **flip-race** difficulty — under perfect play the readers tie regardless of how a Shuffle breaks. But the field study measures a second axis the flip race cannot see: the room between the no-memory floor and the perfect-memory ceiling, which the loop window genuinely widens. That room is the category's **worth**, and it is what the ranking weighs. No player is handed a loop-multiplier; the merit is **earned in play**, banked against the honest floor and ceiling — **GLMP** (merit points), ranked by **GLMR** and headed by the certified **GLM100**.

The loop range earns its place twice: each window [MIN, MAX] is a **distinct Shuffle pool** with its own leaderboard — why the Feast counts them separately — and each carries its own **worth**, the room a memory can convert. The WR tie is revealed for what it is: a **knife-edge**, balanced only under flawless free play, that both the full toolset and real human memory break toward the reader who sees structure. Cyrus wins not because the loops are harder, but because he reads them.

> **In one line: LL sets the room; worth is what weights the categories, and the merit earned inside it is what the ranking rewards.** Strategy and cognition do the rest — and both favour the cycler more sharply as P grows.

References. A389262 (CST) — Grego (Gergely Földvári), 2025. A000041 — partition numbers. A000142 — n!. Golomb & Gaal, *On the Number of Permutations on n Objects with Greatest Cycle Length k*, Adv. Appl. Math. 20 (1998). Foata & Schützenberger, fundamental bijection on permutations. Miller, *The Magical Number Seven*, Psychol. Rev. 63 (1956). Cowan, *The magical number 4 in short-term memory*, Behav. Brain Sci. 24 (2001). Monte-Carlo figures from the project simulations *cyrus_rowen.py* and *full_game.py*.